MECHANISMS AND MECHANICAL DESIGN

(R15A0365)

COURSE FILE

III B. Tech I Semester

(2018-2019)

Prepared By

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Department of Aeronautical Engineering

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

Affiliated to JNTU, Hyderabad, Approved by AICTE - Accredited by NBA & NAAC – 'A' Grade - ISO 9001:2015 Certified)

Maisammaguda, Dhulapally (Post Via. Kompally), Secunderabad – 500100, Telangana State, India.

MRCET VISION

- To become a model institution in the fields of Engineering, Technology and Management.
- To have a perfect synchronization of the ideologies of MRCET with challenging demands of International Pioneering Organizations.

MRCET MISSION

To establish a pedestal for the integral innovation, team spirit, originality and competence in the students, expose them to face the global challenges and become pioneers of Indian vision of modern society.

MRCET QUALITY POLICY.

- To pursue continual improvement of teaching learning process of Undergraduate and Post Graduate programs in Engineering & Management vigorously.
- To provide state of art infrastructure and expertise to impart the quality education.

PROGRAM OUTCOMES (PO's)

Engineering Graduates will be able to:

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design / development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multi disciplinary environments.
12. **Life- long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

DEPARTMENT OF AERONAUTICAL ENGINEERING

VISION

Department of Aeronautical Engineering aims to be indispensable source in Aeronautical Engineering which has a zeal to provide the value driven platform for the students to acquire knowledge and empower themselves to shoulder higher responsibility in building a strong nation.

MISSION

The primary mission of the department is to promote engineering education and research. To strive consistently to provide quality education, keeping in pace with time and technology. Department passions to integrate the intellectual, spiritual, ethical and social development of the students for shaping them into dynamic engineers.

QUALITY POLICY STATEMENT

Impart up-to-date knowledge to the students in Aeronautical area to make them quality engineers. Make the students experience the applications on quality equipment and tools. Provide systems, resources and training opportunities to achieve continuous improvement. Maintain global standards in education, training and services.

PROGRAM EDUCATIONAL OBJECTIVES

Aeronautical Engineering

1. **PEO1 (PROFESSIONALISM & CITIZENSHIP):** To create and sustain a community of learning in which students acquire knowledge and learn to apply it professionally with due consideration for ethical, ecological and economic issues.
2. **PEO2 (TECHNICAL ACCOMPLISHMENTS):** To provide knowledge based services to satisfy the needs of society and the industry by providing hands on experience in various technologies in core field.
3. **PEO3 (INVENTION, INNOVATION AND CREATIVITY):** To make the students to design, experiment, analyze, and interpret in the core field with the help of other multi disciplinary concepts wherever applicable.
4. **PEO4 (PROFESSIONAL DEVELOPMENT):** To educate the students to disseminate research findings with good soft skills and become a successful entrepreneur.
5. **PEO5 (HUMAN RESOURCE DEVELOPMENT):** To graduate the students in building national capabilities in technology, education and research

PROGRAM SPECIFIC OUTCOMES – Aeronautical Engineering

1. To mould students to become a professional with all necessary skills, personality and sound knowledge in basic and advance technological areas.
2. To promote understanding of concepts and develop ability in design manufacture and maintenance of aircraft, aerospace vehicles and associated equipment and develop application capability of the concepts sciences to engineering design and processes.
3. Understanding the current scenario in the field of aeronautics and acquire ability to apply knowledge of engineering, science and mathematics to design and conduct experiments in the field of Aeronautical Engineering.
4. To develop leadership skills in our students necessary to shape the social, intellectual, business and technical worlds.

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

	L	T/P/D	C
III Year B. Tech, ANE-I Sem	**4**	**-/-/-**	**3**

(R15A0365) MECHANISMS AND MECHANICAL DESIGN (OPEN ELECTIVE-III)

Objectives:

- The subject gives in depth knowledge on general mechanisms and mechanical design of which aircraft systems are important component.

UNIT – I

Mechanisms: Elements of links: Classification, Types of kinematic pairs: Lower and higher pairs, closed and open pairs. Constrained motion. Kinematic chain, inversions of mechanisms: inversion of quadratic cycle. Chain – single and double slider crank chains.

UNIT – II

Kinematic Analysis and Design of Mechanisms:

Kinematic analysis: Velocity and acceleration. Motion of link in machine determination of velocity and acceleration diagrams – graphical method. Application of relative velocity method for four bar chain. Analysis of slider crank chain for displacement, velocity and acceleration of sliding- Acceleration diagram for a given mechanism, Klein's construction, Coriolis acceleration, Determination of Coriolis component of acceleration.

Instantaneous centre of rotation, centroids and axodes – relative motion between two bodies – Three centres in line theorem – Graphical determination of instantaneous centre, diagrams for simple mechanisms and determination of angular velocity of points and links.

Kinematic Design: Four bar mechanism, Freudenstein equation. Precession point synthesis, Chebyshev`s method, structural error.

UNIT – III

Gyroscope – Processional Motion: The gyroscope – free and restrained – working principle – the free gyro, rate gyro, integrating gyro as motion measuring instruments. Effect of precession on the stability of vehicles – motorbikes, automobiles, airplanes and ships, Static and dynamic forces generated due to in precession in rotating mechanisms.

UNIT – IV

CAMS and Followers: Cams and followers – definition, uses – types – terminology. Types of follower motion – uniform velocity, simple harmonic motion and uniform acceleration. Maximum velocity and acceleration during outward and return strokes. Roller follower, circular cam with straight, concave and convex flanks.

UNIT – V

Gears and Gear Trains: Introduction to gears – types , law of gearing. Tooth profiles – specifications, classification – helical, bevel and worm gears, simple and reverted gear train, epicyclic gear trains – velocity ratio or train value.

Text Books:

1. The Theory of machines – Thomas Beven., Third Edition – Pearson Publishers.
2. Theory of machines and Mechaisms Third Edition – John J. Uicker, Jr. Gordon R. Pennock, Josph E. Shigley, Oxford Publisher.

Reference Books:

1. Mechanism and Machine Theory – J. S Rao, R.V.D Dukkipati, New age Publishers.
2. Theory of Machines, - III rd Edition Sadhu Singh, Pearson Publishers.

Outcomes:

- Application of principles in the formation of mechanisms and their kinematics.
- Able to understand the effect of friction in different machine elements.
- Can analyze the forces and toques acting on simple mechanical systems

LESSON PLANNER

DEPARTMENT OF AERONAUTICAL ENGINEERING

SUB: MECHANISMS & MECHANICAL DESIGN **SEMMESTER – III Year I Sem.**
FACULTY: S. SHAILESH BABU, Asst PROF

Objectives:

The subject gives in depth knowledge on general mechanisms and mechanical design of which aircraft systems are important component.

S.no	UNIT NO	TOPICS PLANNED TO COVER	NO OF CLASSES
1	**Mechanisms**	Elements of links: Classification	3
		Types of kinematic pairs: Lower and higher pairs, closed and open pairs	3
		Constrained motion. Kinematic chain	2
		inversions of mechanisms: inversion of quadratic cycle	2
		Chain – single and double slider crank chains	3
		Unit 1 Total no of classes	**13**
2	**Kinematic Analysis and Design of Mechanisms**	**Kinematic analysis:** Velocity and acceleration. Motion of link in machine determination of velocity and acceleration diagrams – graphical method	3
		Application of relative velocity method for four bar chain	1
		Analysis of slider crank chain for displacement, velocity and acceleration of sliding- Acceleration diagram for a given mechanism	3
		Klein's construction, Coriolis acceleration	1
		Determination of Coriolis component of acceleration	1
		Instantaneous centre of rotation, centroids and axodes – relative motion between two bodies	1
		Three centres in line theorem – Graphical determination of instantaneous centre	1
		Diagrams for simple mechanisms and determination of angular velocity of points and links	2
		Kinematic Design: Four bar mechanism, Freudenstein equation	1
		Precession point synthesis, Chebyshev`s method, structural error.	2
		Unit 2 Total no of classes	**16**
3	**Gyroscope**	**Processional Motion:** The gyroscope – free and restrained – working principle	1
		the free gyro, rate gyro, integrating gyro as motion measuring instruments	2
		Effect of precession on the stability of vehicles – motorbikes, automobiles, airplanes and ships	6
		Static and dynamic forces generated due to in precession in rotating mechanisms	3
		Unit 3 Total no of classes	**12**

4	**CAMS and Followers**	Cams and followers – definition, uses – types – terminology. Types of follower motion	2
		uniform velocity, simple harmonic motion and uniform acceleration	2
		Maximum velocity and acceleration during outward and return strokes	2
		Roller follower, circular cam with straight	2
		concave and convex flanks	2
		Unit 4 Total no of classes	**10**
5	**Gears and Gear Trains**	Introduction to gears – types	1
		Law of gearing	1
		Tooth profiles – specifications, classification – helical, bevel and worm gears	3
		simple and reverted gear train	2
		epicyclic gear trains	1
		velocity ratio or train value	2
		Unit 5 Total no of classes	**10**
		TOTAL NO OF CLASSES FOR COURSE	**61**

Text Books:

3. The Theory of machines – Thomas Beven., Third Edition – Pearson Publishers.
4. Theory of machines and Mechaisms Third Edition – John J. Uicker, Jr. Gordon R. Pennock, Josph E. Shigley, Oxford Publisher.

Reference Books:

3. Mechanism and Machine Theory – J. S Rao, R.V.D Dukkipati, New age Publishers.
4. Theory of Machines, - III rd Edition Sadhu Singh, Pearson Publishers.

Outcomes:

- Application of principles in the formation of mechanisms and their kinematics.
- Able to understand the effect of friction in different machine elements.
- Can analyze the forces and toques acting on simple mechanical systems

UNIT – I

MECHANISM

When one of the links of a kinematic chain is fixed, the chain is known as ***mechanism***. It may be used for transmitting or transforming motion

A mechanism with four links is known as simple mechanism, and the mechanism with more than four links is known as compound mechanism. When a mechanism is required to transmit power or to do some particular type of work, it then becomes a machine. In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely.

A little consideration will show that a mechanism may be regarded as a machine in which each part is reduced to the simplest form to transmit the required motion.

ELEMENTS OF LINKS:

A machine is a device that is capable of converting the available forms of energy to useful work. Each part of a machine, that undergoes relative motion with respect to some other part, is called kinematic link (or kinematic element). Kinematic links help in the transmission of motion, from one machine part to another.

Types of Kinematic links:

Based on rigidity, kinematic links can be broadly classified into three types. They are:

1. Rigid link
2. Flexible link and
3. Fluid link

Rigid Link:

Rigid links are those kinematic links that do not undergo any change of shape when transmitting motion (or when subjected to external forces). In reality, no rigid links exist. However, kinematic links whose deformation is very small are considered as rigid links. Some good examples of rigid links are crankshafts, connecting rods and cam followers. These links do not undergo significant deformation while transmitting motion.

Flexible link:

A flexible link is a resistant kinematic link that undergoes partial deformation when transmitting motion. Its deformation does not hinder its effectiveness of transmission. Some examples of flexible links are belts (in belt drives) and chains (in chain drives).

The belt found in the following image is a flexible link.

Fluid Link:

A fluid link makes use of a fluid (liquid or gas) to transmit motion, by means of pressure. Fluid links always undergo deformation when transmitting motion. Some good examples where fluid links are used are pneumatic punching presses, hydraulic jacks and hydraulic brakes.

KINEMATIC PAIR:

The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (*i.e.* in a definite direction), the pair is known as ***kinematic pair***.

CLASSIFICATION OF KINEMATIC PAIRS:

The kinematic pairs may be classified according to the following considerations:

1. ***According to the type of relative motion between the elements.***

(***a***) ***Sliding pair.*** When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. The piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed etc. are the examples of a sliding pair. A little consideration will show that a sliding pair has a completely constrained motion.

(***b***) ***Turning pair***. When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine, lathe spindle supported in head stock, cycle wheels turning over their axles etc. are the examples of a turning pair. A turning pair also has a completely constrained motion.

(***c***) ***Rolling pair***. When the two elements of a pair are connected in such a way that one roll over another fixed link, the pair is known as rolling pair. Ball and roller bearings are examples of rolling pair.

(***d***) ***Screw pair.*** When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut, and bolt with a nut are examples of a screw pair.

(***e***) ***Spherical pair***. When the two elements of a pair are connected in such a way that one element (with spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair. The ball and socket joint, attachment of a car mirror, pen stand etc., are the examples of a spherical pair.

2. ***According to the type of contact between the elements***.

(***a***) ***Lower pair***. When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as lower pair. It will be seen that sliding pairs, turning pairs and screw pairs form lower pairs.

(***b***) ***Higher pair***. When the two elements of a pair have a line or point contact when relative motion takes place and the motion between the two elements is partly turning and partly sliding, then the pair is known as higher pair. A pair of friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs.

3. ***According to the type of closure***.

(***a***) ***Self closed pair***. When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self closed pair. The lower pairs are self closed pair.

(***b***) ***Force - closed pair***. When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

TYPES OF CONSTRAINED MOTIONS

There are three types of constrained motions:

1. Completely constrained motion
2. Incompletely constrained motion
3. Successfully constrained motion.

Completely constrained motion: When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a **completely constrained motion**. For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (*i.e.* it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank. The motion of a square bar in a square hole, and the motion of a shaft with collars at each end in a circular hole, in the below fig.1 are also examples of completely constrained motion.

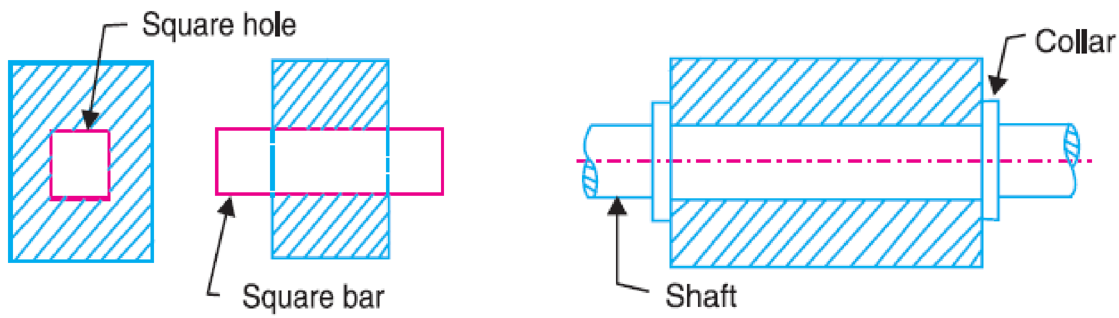


Fig 1: Examples for Completely constrained motion

Incompletely constrained motion: When the motion between a pair can take place in more than one direction, then the motion is called an **incompletely constrained motion**. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig.2, is an example of an incompletely constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other.

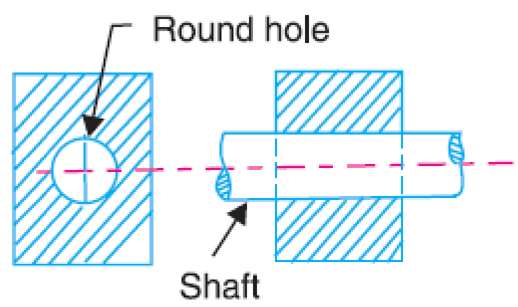


Fig 2: Example for incompletely constrained motion

Successfully constrained motion: When the motion between the elements, forming a pair is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig. 3. The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine valve (these are kept on their seat by a spring) and the piston reciprocating inside an engine cylinder are also the examples of successfully constrained motion.

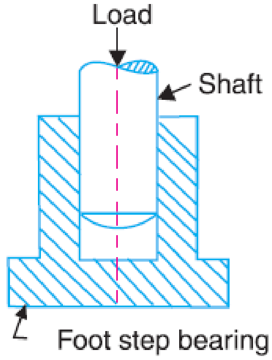


Fig 3: Example for successfully constrained motion

KINEMATIC CHAIN

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (*i.e.* completely or successfully constrained motion), it is called a **kinematic chain.**

(OR)

A kinematic chain may be defined as a combination of kinematic pairs, joined in such a way that each link forms a part of two pairs and the relative motion between the links or elements is completely or successfully constrained.

For example, the crankshaft of an engine forms a kinematic pair with the bearings which are fixed in a pair, the connecting rod with the crank forms a second kinematic pair, the piston with the connecting rod forms a third pair and the piston with the cylinder forms a fourth pair. The total combination of these links is a kinematic chain. If each link is assumed to form two pairs with two adjacent links, then the relation between the number of pairs (p) forming a kinematic chain and the number of links (l) may be expressed in the form of an equation

$$l = 2p - 4 \qquad \ldots \text{(i)}$$

Another relation between the number of links (l) and the number of joints (j) which constitute a kinematic chain is given by the expression

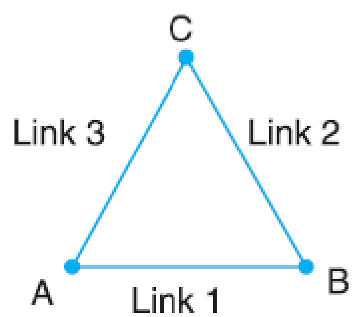
$$j = \frac{3}{2}l - 2 \qquad \ldots \text{(ii)}$$

Let us apply the above equations to the following cases to determine whether each of them is a kinematic chain or not.

1. Consider the arrangement of three links *AB*, *BC* and *CA* with pin joints at *A*, *B* and *C* as shown in Fig. In this case,

   Number of links $l = 3$
   Number of pairs $p = 3$
   Number of joints $j = 3$

   From the equation $l = 2p - 4$
   $3 = 2 \times 3 - 4 = 2$



i.e. L.H.S. > R. H. S.

Now from the equation $j = \frac{3}{2}l - 2$

$$j = \frac{3}{2}x3 - 2 = 2.5$$

i.e. L.H.S. > R. H. S.

Since the arrangement of three links, as shown in Fig. 5.6, does not satisfy the two equations and L.H.S. > R. H. S., therefore it is not a kinematic chain and hence no relative motion is possible. Such type of chain is called ***locked chain*** and forms a rigid frame or structure which is used in bridges and trusses.

2. Consider the arrangement of four links *AB*, *BC*, *CD* and *DA* as shown in Fig. In this case $l = 4$, $p = 4$, and $j = 4$

   From equation **(*i*),** $l = 2p - 4$

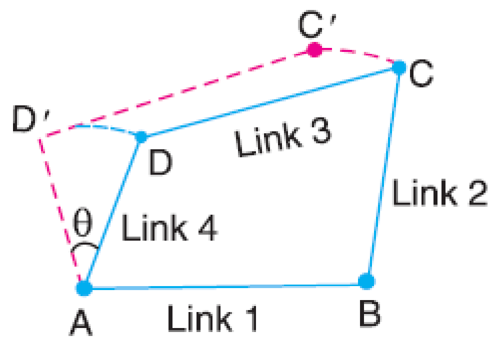
   $4 = 2 \times 4 - 4 = 4$

   i.e. L.H.S. = R.H.S.

   From equation **(*ii*),** $j = \frac{3}{2}l - 2$

   $4 = \frac{3}{2}x4 - 2 = 4$

   i.e. L.H.S. = R.H.S.



   Since the arrangement of four links satisfies the equations **(*i*)** and **(*ii*),** therefore it is a ***kinematic chain of one degree of freedom***.

   A chain in which a single link such as *AD* in Fig. is sufficient to define the position of all other links, it is then called a kinematic chain of one degree of freedom.

   A little consideration will show that in Fig., if a definite displacement (say θ) is given to the link *AD*, keeping the link *AB* fixed, then the resulting displacements of the remaining two links *BC* and *CD* are also perfectly definite. Thus we see that in a four bar chain, the relative motion is completely constrained. Hence it may be called as a ***constrained kinematic chain***, and it is the basis of all machines.

3. Consider an arrangement of five links, as shown in Fig. In this case,

   $l = 5$, $p = 5$, and $j = 5$

   From equation **(*i*),**

   $l = 2p - 4$ or $5 = 2 \times 5 - 4 = 6$

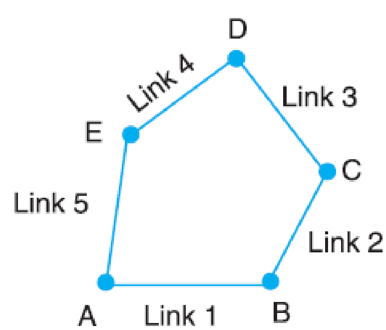
   i.e. L.H.S. < R.H.S.

   From equation **(*ii*),**

   $j = \frac{3}{2}l - 2$

$5 = \frac{3}{2}\ x\ 5 - 2 = 5.5$

i.e. L.H.S. < R.H.S.



Since the arrangement of five links, does not satisfy the equations and L.H.S. < R.H.S. therefore it is not a kinematic chain. Such a type of chain is called ***unconstrained chain*** *i.e*. the relative motion is not completely constrained. This type of chain is of little practical importance.

4. Consider an arrangement of six links, as shown in Fig.. This chain is formed by adding two more links in such a way that these two links form a pair with the existing links as well as form themselves a pair. In this case

   $l = 6$, $p = 5$, and $j = 7$

   From equation **(*i*),**

   $l = 2p - 4$ or $6 = 2 \times 5 - 4 = 6$

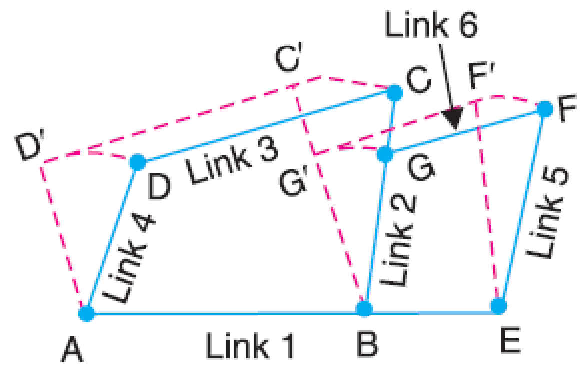
   i.e. L.H.S. = R.H.S.

   From equation **(*ii*),**

   $$j = \frac{3}{2}l - 2$$

   $7 = \frac{3}{2} x\, 6 - 2 = 7$

   i.e. L.H.S. = R.H.S.

   Since the arrangement of six links, as shown in Fig. satisfies the equations (*i.e.* L.H.S. = R.H.S.), therefore it is a kinematic chain.

Note: A chain having more than four links is known as **compound kinematic chain.**

MECHANISM

When one of the links of a kinematic chain is fixed, the chain is known as ***Mechanism***. It may be used for transmitting or transforming motion

A mechanism with four links is known as ***simple mechanism***, and the mechanism with more than four links is known as ***compound mechanism***. When a mechanism is required to transmit power or to do some particular type of work, it then becomes a ***machine***. In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely.

Inversion of Mechanism

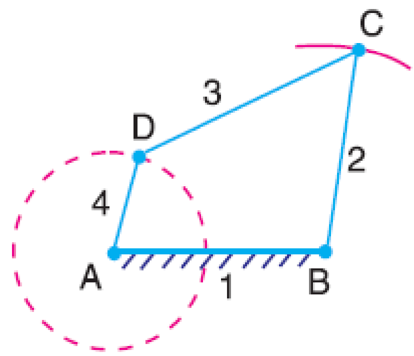
We have already discussed that when one of links is fixed in a kinematic chain, it is called a mechanism. So we can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanisms by fixing different links in a kinematic chain is known as ***inversion of the mechanism***. It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to the fixed link) may be changed drastically.

TYPES OF KINEMATIC CHAINS

The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view:

1. Four bar chain or quadric cyclic chain,
2. Single slider crank chain, and
3. Double slider crank chain.

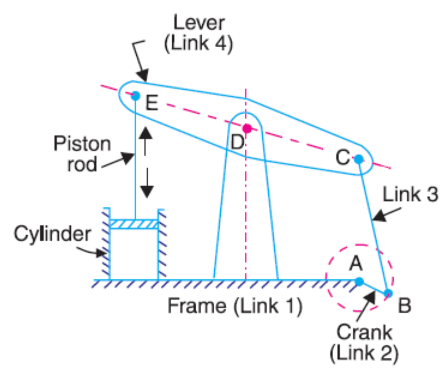
FOUR BAR CHAIN OR QUADRIC CYCLE CHAIN:



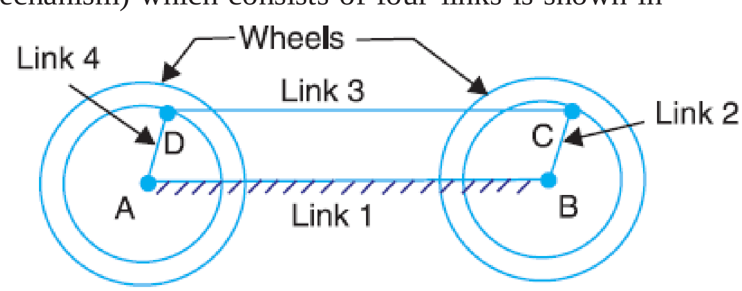
The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain. It consists of four links, each of them forms a turning pair at *A, B, C* and *D*. The four links may be of different lengths. According to **Grashof 's law** for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.
A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof 's law. Such a link is known as ***crank*** or ***driver***. In Fig. *AD* (link 4) is a crank. The link *BC* (link 2) which makes a partial rotation or oscillates is known as ***lever*** or ***rocker*** or ***follower*** and the link *CD* (link 3) which connects the crank and lever is called ***connecting rod*** or ***coupler.*** The fixed link *AB* (link 1) is known as ***frame*** of the mechanism. When the crank (link 4) is the driver, the mechanism is transforming rotary motion into oscillating motion.

INVERSIONS OF FOUR BAR CHAIN

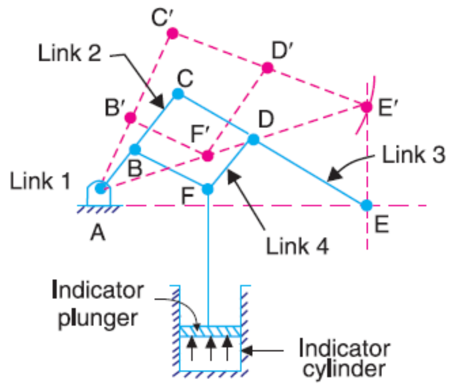
***Beam engine* (*crank and lever mechanism*):** A part of the mechanism of a beam engine (also known as crank and lever mechanism) which consists of four links, is shown in Fig. In this mechanism, when the crank rotates about the fixed centre *A*, the lever oscillates about a fixed centre *D*. The end *E* of the lever *CDE* is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.



***Coupling rod of a locomotive* (*Double crank mechanism*):** The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links is shown in Fig. In this mechanism, the links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.



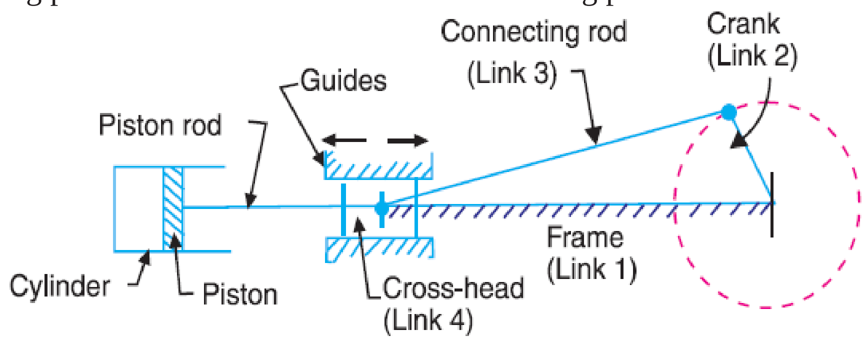
***Watt's indicator mechanism* (*Double lever mechanism*):** A Watt's indicator mechanism also known as Watt's straight line mechanism or double lever mechanism which consists of four links, is shown in Fig. The four links are: fixed link at *A*, link *AC*, link *CE* and link *BFD*. It may be noted that *BF* and *FD* form one link because these two parts have no relative motion between them. The links CE and BFD act as levers. The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line. The initial position of the mechanism is shown in Fig. by full lines whereas the dotted lines show the position of the mechanism when the gas or steam pressure acts on the indicator plunger.



SINGLE SLIDER CRANK CHAIN

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pairs. It is, usually, found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion and vice versa.

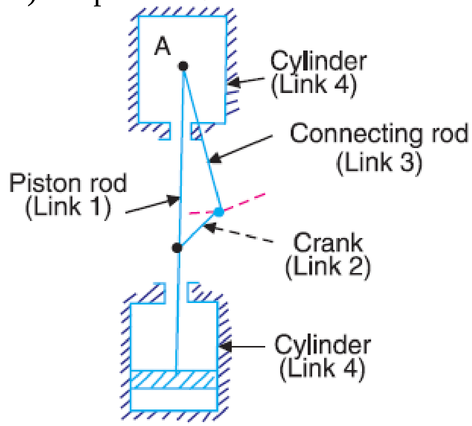
In a single slider crank chain, as shown in Fig, the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.



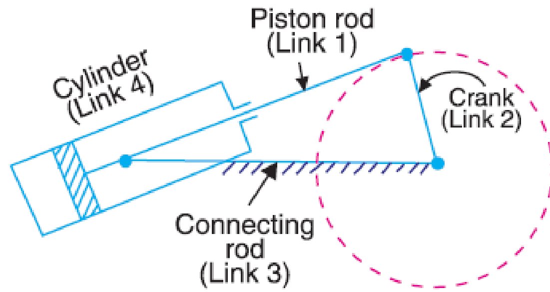
The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank; link 3 corresponds to the connecting rod and link 4 corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

INVERSIONS OF SINGLE SLIDER CRANK CHAIN

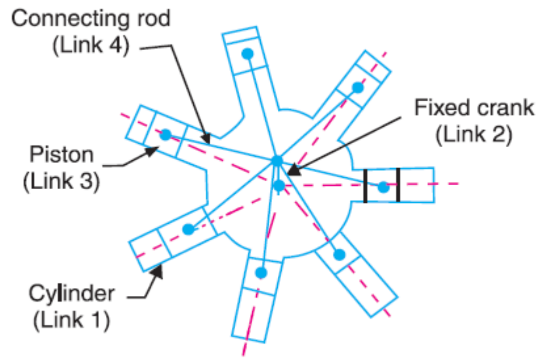
Pendulum pump or Bull engine: In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (*i.e.* sliding pair), as shown in Fig. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates.



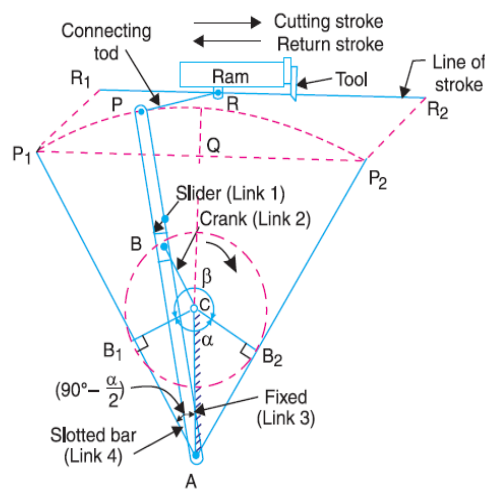
Oscillating cylinder engine: The arrangement of oscillating cylinder engine mechanism, as shown in Fig. is used to convert reciprocating motion into rotary motion. In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at *A*.



Rotary internal combustion engine or Gnome engine: Sometimes back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines are used in its place. It consists of seven cylinders in one plane and all revolves about fixed centre *D*, as shown in Fig. 5 while the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.



Crank and slotted lever quick return motion mechanism: This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines. In this mechanism, the link *AC* (*i.e.* link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank *CB* revolves with uniform angular speed about the fixed centre *C*. A sliding block attached to the crank pin at *B* slides along the slotted bar *AP* and thus causes *AP* to oscillate about the pivoted point *A*. A short link *PR* transmits the motion from *AP* to the ram which carries the tool and reciprocates along the line of stroke R_1R_2. The line of stroke of the ram (*i.e.* R_1R_2) is perpendicular to *AC* produced.



In the extreme positions, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB_2 to CB_1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^o - \beta} \; or \; \frac{360^o - \alpha}{\alpha}$$

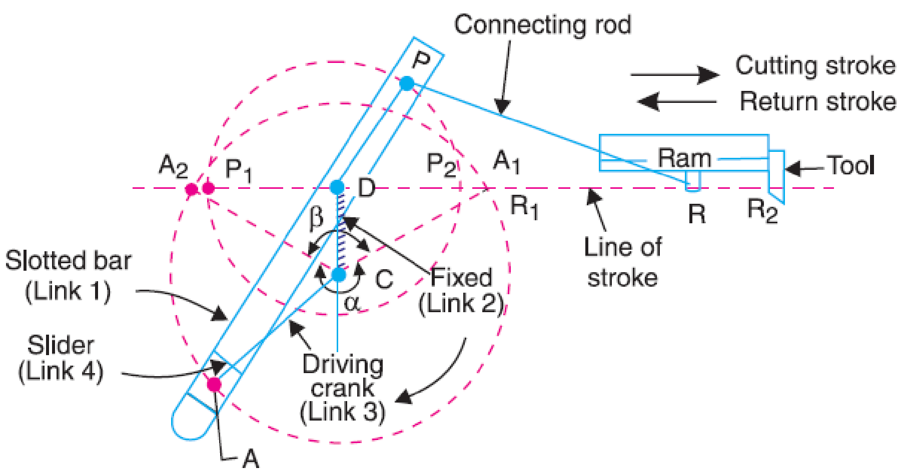
Since the tool travels a distance of *R*1 *R*2 during cutting and return stroke, therefore travel of the tool or length of stroke

$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin\angle P_1AQ$$

$$= 2AP_1 sin\left(90^0 - \frac{\alpha}{2}\right) = 2APcos\ \frac{\alpha}{2}$$
$$= 2AP \text{ x } \frac{CB_1}{AC}$$
$$= 2AP \text{ x } \frac{CB}{AC}$$

NOTE: From Fig. we see that the angle β made by the forward or cutting stroke is greater than the angle α described by the return stroke. Since the crank rotates with uniform angular speed, therefore the return stroke is completed within shorter time. Thus it is called quick return motion mechanism.

Whitworth quick return motion mechanism: This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link *CD* (link 2) forming the turning pair is fixed, as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank *CA* (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at *A* slides along the slotted bar *PA* (link 1) which oscillates at a pivoted point *D*. The connecting rod *PR* carries the ram at *R* to which a cutting tool is fixed. The motion of the tool is constrained along the line *RD* produced, *i.e*. along a line passing through *D* and perpendicular to *CD*.



When the driving, crank *CA* moves from the position CA_1 to CA_2 (or the link *DP* from the position DP_1 to DP_2) through an angle α in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance 2 *PD*.

Now when the driving crank moves from the position CA_2 to CA_1 (or the link *DP* from DP_2 to DP_1) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram (*i.e*. during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA_1 to CA_2. Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from CA_2 to CA_1.

Since the crank link *CA* rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke.

The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^o - \alpha} \; or \; \frac{360^o - \beta}{\beta}$$

NOTE In order to find the length of effective stroke $R_1 R_2$, mark $P_1 R_1 = P_2 R_2 = PR$. The length of effective stroke is also equal to 2 PD.

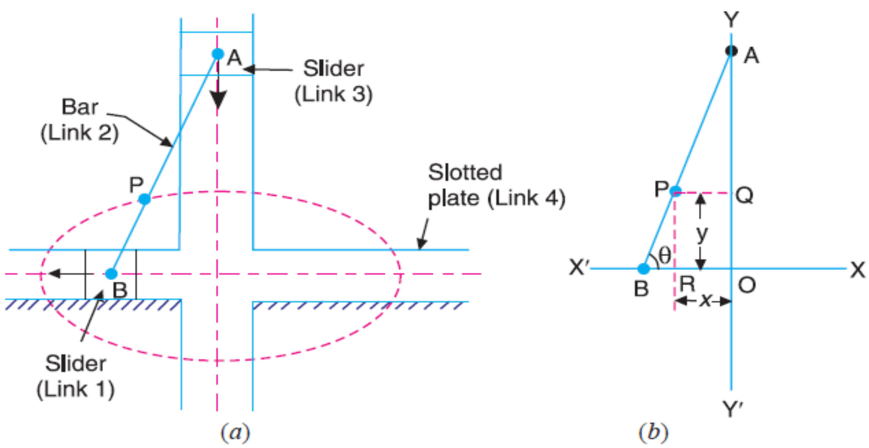
DOUBLE SLIDER CRANK CHAIN

A kinematic chain which consists of two turning pairs and two sliding pairs is known as ***double slider crank chain***, as shown in Fig. We see that the link 2 and link 1 form one turning pair and link 2 and link 3 form the second turning pair. The link 3 and link 4 form one sliding pair and link 1 and link 4 form the second sliding pair.

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

Elliptical trammels. It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shown in Fig. The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3 are known as sliders and form sliding pairs with link 4. The link AB (link 2) is a bar which forms turning pair with links 1 and 3.

When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as P traces out an ellipse on the surface of link 4, as shown in Fig. (a). A little consideration will show that AP and BP are the semi-major axis and semi-minor axis of the ellipse respectively.



Let us take OX and OY as horizontal and vertical axes and let the link BA is inclined at an angle θ with the horizontal, as shown in Fig. (b). Now the co-ordinates of point P on the link BA will be

$$x = PQ = AP \cos\theta; \text{ and } y = PR = BP \sin\theta$$

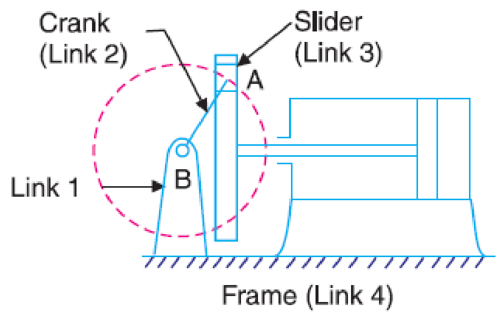
$$\frac{x}{AP} = cos\theta \; and \; \frac{y}{BP} = sin\theta$$

Squaring and adding,

$$\frac{x^2}{AP^2} + \frac{Y^2}{BP^2} = cos^2\theta + sin^2\theta = 1$$

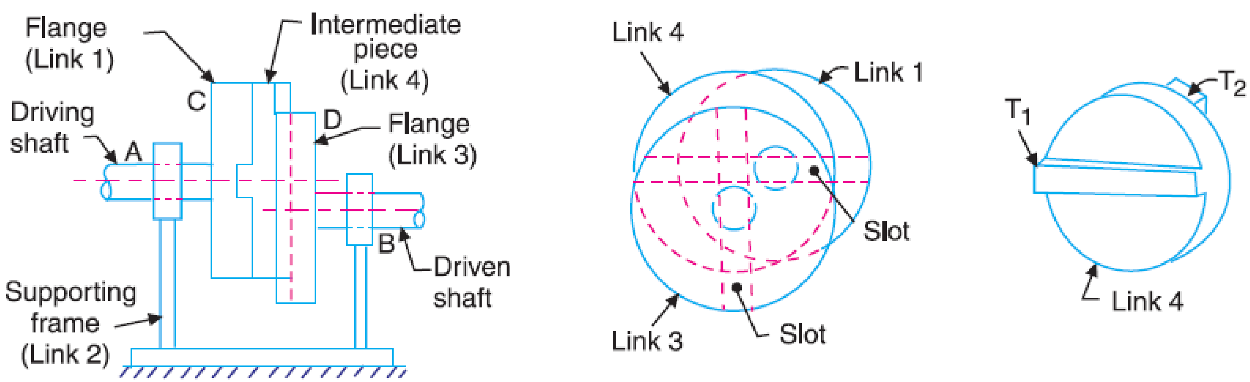
This is the equation of an ellipse. Hence the path traced by point P is an ellipse whose semi major axis is AP and semi-minor axis is BP.

Scotch yoke mechanism. This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In Fig. link 1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about *B* as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.



Oldham's coupling. An Oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is obtained by fixing the link 2, as shown in Fig. (*a*). The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.

The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut in their inner faces, as shown in Fig. (*b*). The intermediate piece (link 4) which is a circular disc, have two tongues (*i.e.* diametrical projections) T_1 and T_2 on each face at right angles to each other, as shown in Fig. (*c*). The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3). The link 4 can slide or reciprocate in the slots in the flanges.



(*a*) (*b*) (*c*)

When the driving shaft *A* is rotated, the flange *C* (link 1) causes the intermediate piece (link 4) to rotate at the same angle through which the flange has rotated, and it further rotates the flange *D* (link 3) at the same angle and thus the shaft *B* rotates. Hence links 1, 3 and 4 have the same angular velocity at every instant. A little consideration will show that there is a sliding motion between the link 4 and each of the other links 1 and 3.

If the distance between the axes of the shafts is constant, the centre of intermediate piece will describe a circle of radius equal to the distance between the axes of the two shafts. Therefore, the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the centre of the disc along its circular path.

Let ω = Angular velocity of each shaft in rad/s, and

r = Distance between the axes of the shafts in metres.

$\therefore$ Maximum sliding speed of each tongue (in m/s),

$$v = \omega . r$$

UNIT – II
KINEMATIC ANALYSIS AND DESIGN OF MECHANISMS

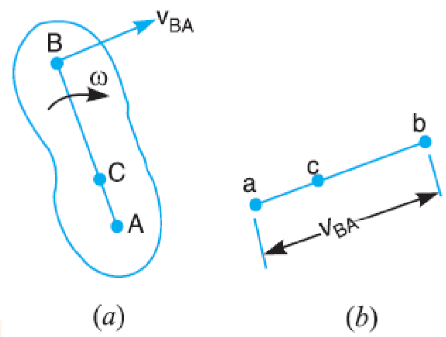
VELOCITY: The rate of change of displacement is known as Velocity.
If the displacement is linear, then it is called as linear velocity (v) and if the displacement is angular, then it is called as Angular velocity (ω).

Linear velocity, $v = \frac{ds}{dt}$ where ds is change in linear displacement

Angular velocity, $\omega = \frac{d\theta}{dt}$ where is change in linear displacement

Relative Velocity of Two Bodies Moving in Straight Lines

Here we shall discuss the application of vectors for the relative velocity of two bodies moving along parallel lines and inclined lines, as shown in Fig 7.1(a) and 7.2 (a)respectively. Consider two bodies A and B moving along parallel lines in the same direction with absolute velocities v_A and v_B such that $v_A > v_B$, as shown in Fig. 1 (a). The relative velocity of A with respect to B,



$$v_{AB} = \text{Vector difference of } v_A \text{ and } v_B = \overline{v_A} -$$

From Fig. 1 (b), the relative velocity of A with respect to B (i.e. v_{AB}) may be written in the vector form as follows:

$$\overline{ba} = \overline{oa} - \overline{ob}$$

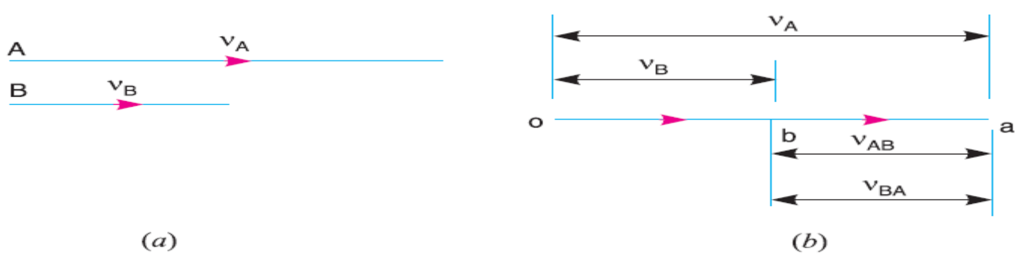


Fig. 7.1. Relative velocity of two bodies moving along parallel lines.

Similarly, the relative velocity of *B* with respect to *A* ,

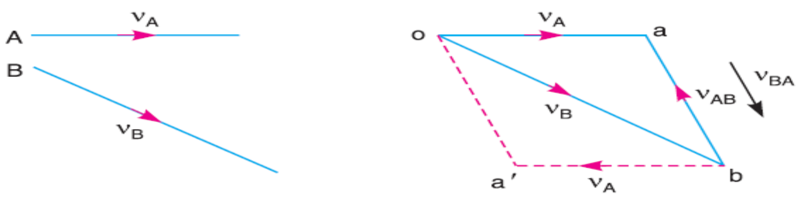
$$v_{BA} = \text{Vector difference of } v_B \text{ and } v_A = \overline{v_B} - \overline{v_A} \quad ...(ii)$$

$$\overline{ab} = \overline{ob} - \overline{oa}$$

Now consider the body B moving in an inclined direction as shown in Fig. 7.2 (a). The relative velocity of A with respect to B may be obtained by the law of parallelogram of velocities or triangle law of velocities. Take any fixed point o and draw vector oa to represent v_A in magnitude and direction to some suitable scale. Similarly, draw vector ob to represent v_B in magnitude and direction to the same scale. Then vector ba represents the relative velocity of A with respect to B as shown in Fig. 7.2 (b). In the similar way as discussed above, the relative velocity of A with respect to B,

$$v_{AB} = \text{Vector difference of } v_A \text{ and } v_B = \overline{v_A} - \overline{v_B}$$

$$\overline{ba} = \overline{oa} - \overline{ob}$$



(a) (b)

Fig. 7.2. Relative velocity of two bodies moving along inclined lines.

Similarly, the relative velocity of B with respect to A,

$$v_{BA} = \text{Vector difference of } v_B \text{ and } v_A = \overline{v_B} - \overline{v_A}$$

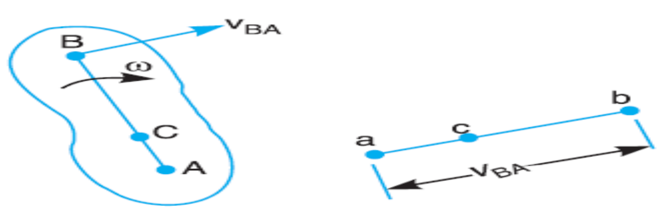
$$\overline{ab} = \overline{ob} - \overline{oa}$$

From above, we conclude that the relative velocity of point A with respect to B (v_{AB}) and the relative velocity of point B with respect A (v_{BA}) are equal in magnitude but opposite in direction, i.e.

$$v_{AB} = -v_{BA} \quad \text{or} \quad \overline{ba} = -\overline{ab}$$

Motion of a Link

Consider two points A and B on a rigid link AB, as shown in Fig. (a). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB. Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.



(a) (b)

The relative velocity of B with respect to A (i.e. v_{BA}) is represented by the vector ab and is perpendicular to the line AB as shown in Fig(b).

Let $\qquad \omega$ = Angular velocity of the link AB about A.

We know that the velocity of the point B with respect to A,

$$v_{BA} = \overline{ab} = \omega . AB \qquad \text{...}(i)$$

Similarly, the velocity of any point C on AB with respect to A,

$$v_{CA} = \overline{ac} = \omega . AC \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*),

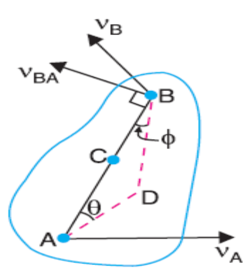
$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB} \qquad \text{...}(iii)$$

Thus, we see from equation (iii), that the point c on the vector ab divides it in the same ratio as C divides the link AB.

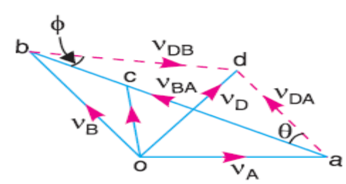
Velocity of a Point on a Link by Relative Velocity Method

Consider two points A and B on a link as shown in Fig. 4 (a). Let the absolute velocity of the point A i.e. v_A is known in magnitude and direction and the absolute velocity of the point B i.e. v_B is known in direction only. Then the velocity of B may be determined by drawing the velocity diagram as shown in Fig (b). The velocity diagram is drawn as follows:

1. Take some convenient point o, known as the pole.
2. Through o, draw oa parallel and equal to v_A, to some suitable scale.
3. Through a, draw a line perpendicular to AB of Fig. 4 (a). This line will represent the velocity of B with respect to A, i.e. v_{BA}.
4. Through o, draw a line parallel to v_B intersecting the line of v_{BA} at b.
5. Measure ob, which gives the required velocity of point B (v_B), to the scale.



(*a*) Motion of points on a link.



(*b*) Velocity diagram.

Velocities in Slider Crank Mechanism

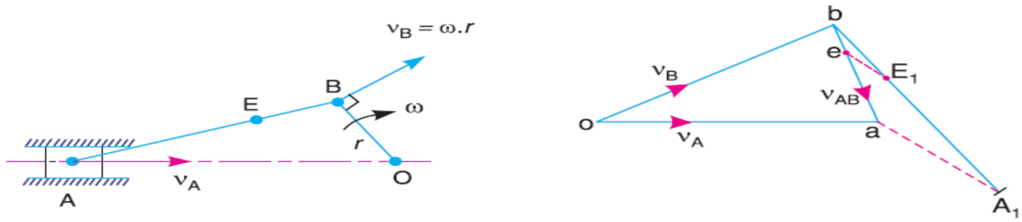
In the previous article, we have discussed the relative velocity method for the velocity of anypoint on a link, whose direction of motion and velocity of some other point on the same link is known.

The same method may also be applied for the velocities in a slider crank mechanism.

A slider crank mechanism is shown in Fig. 7.5 (a). The slider A is attached to the connecting rod AB. Let the radius of crank OB be r and let it rotates in a clockwise direction, about the point O with uniform angular velocity ω rad/s. Therefore, the velocity of B i.e. v_B is known in magnitude and direction. The slider reciprocates along the line of stroke AO.

The velocity of the slider A (i.e. v_A) may be determined by relative velocity method as discussed below:

1. From any point o, draw vector ob parallel to the direction of vB (or perpendicular to OB) such that ob = v_B = ω .r, to some suitable scale, as shown in Fig. 5 (b).



(a) Slider crank mechanism. (b) Velocity diagram.

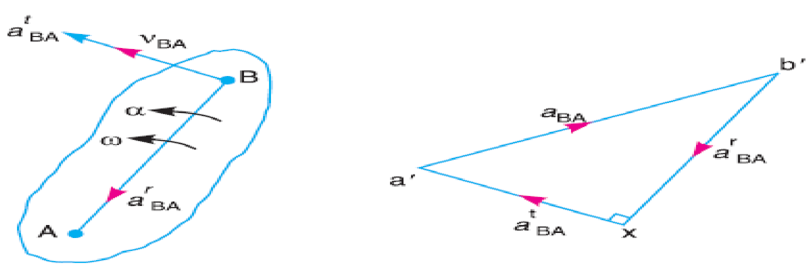
2. Since AB is a rigid link, therefore the velocity of A relative to B is perpendicular to AB. Now draw vector ba perpendicular to AB to represent the velocity of A with respect to B i.e. v_{AB}.
3. From point o, draw vector oa parallel to the path of motion of the slider A (which is along AO only). The vectors ba and oa intersect at a. Now oa represents the velocity of the slider A i.e. v_A, to the scale.

The angular velocity of the connecting rod AB (ω_{AB}) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB} \qquad \text{(Anticlockwise about A)}$$

Acceleration Diagram for a Link

Consider two points A and B on a rigid link as shown in Fig (a). Let the point B moves with respect to A, with an angular velocity of ω rad/s and let α rad/s2 be the angular acceleration of the link AB.



(a) Link. (b) Acceleration diagram.

We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components:

1. The centripetal or radial component, which is perpendicular to the velocity of the particle at the given instant.
2. The tangential component, which is parallel to the velocity of the particle at the given instant. Thus for a link AB, the velocity of point B with respect to A (i.e. v_{BA}) is perpendicular to the link AB as

shown in Fig (a). Since the point B moves with respect to A with an angular velocity of ω rad/s, therefore centripetal or radial component of the acceleration of B with respect to A,

$$a^r_{BA} = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v^2_{BA} / AB \qquad \ldots\left(\because \omega = \frac{v_{BA}}{AB}\right)$$

This radial component of acceleration acts perpendicular to the velocity v_{BA}, In other words, it acts parallel to the link AB.

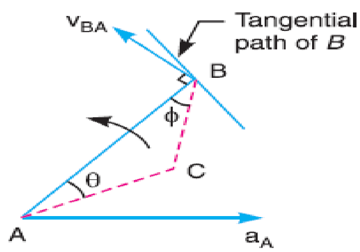
We know that tangential component of the acceleration of B with respect to A,

$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

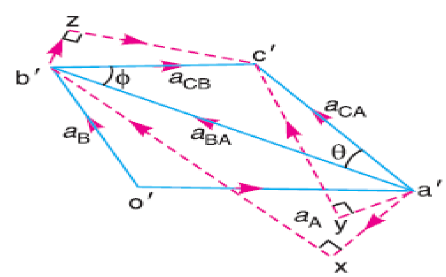
This tangential component of acceleration acts parallel to the velocity v_{BA}. In other words, it acts perpendicular to the link AB.

In order to draw the acceleration diagram for a link *A B*, as shown in Fig. 8.1 (*b*), from any point *b'*, draw vector ***b'x parallel to BA*** to represent the radial component of acceleration of *B* with respect to *A i.e.* a^r_{BA} and from point *x* draw vector *xa'* perpendicular to *B A* to represent the tangential component of acceleration of *B* with respect to *A i.e.* a^t_{BA} . ***Join b' a'.*** The vector *b' a'* (known as ***acceleration image*** of the link *A B*) represents the total acceleration of *B* with respect to *A* (*i.e.* a_{BA}) and it is the vector sum of radial component (a^r_{BA}) and tangential component (a^t_{BA}) of acceleration.

Acceleration of a Point on a Link



(*a*) Points on a Link.



(*b*) Acceleration diagram.

Consider two points A and B on the rigid link, as shown in Fig(a). Let the acceleration of the point A i.e. a_A is known in magnitude and direction and the direction of path of B is given. The acceleration of the point B is determined in magnitude and direction by drawing the acceleration diagram as discussed below.

1. From any point o', draw vector o'a' parallel to the direction of absolute acceleration at point A i.e. a_A, to some suitable scale, as shown in Fig. (b).

2. We know that the acceleration of B with respect to A i.e. a_{BA} has the following two components:

(i) Radial component of the acceleration of B with respect to A i.e a^r_{BA} , and

(ii) Tangential component of the acceleration B with respect to A i.e. a^t_{BA} These two components are mutually perpendicular.

3. Draw vector a'x parallel to the link AB (because radial component of the acceleration of B with respect to A will pass through AB), such that

$$\text{vector } a'x = a^r_{BA} = v^2_{BA} / AB$$

4. From point x, draw vector xb' perpendicular to AB or vector a'x (because tangential component of B with respect to A i.e. at BA, is perpendicular to radial component a^r_{BA}) and through o' draw a line parallel to the path of B to represent the absolute acceleration of B i.e. a_B. The vectors xb' and o' b' intersect at b'. Now the values of a_B and a^t_{BA} may be measured, to the scale.

5. By joining the points a' and b' we may determine the total acceleration of B with respect to A *i.e.* a_{BA}. The vector $a'\,b'$ is known as ***acceleration image*** of the link $A\,B$.

6. For any other point C on the link, draw triangle $a'\,b'\,c'$ similar to triangle ABC. Now vector $b'\,c'$ represents the acceleration of C with respect to B *i.e.* a_{CB}, and vector $a'\,c'$ represents the acceleration of C with respect *to A i.e.* a_{CA}. As discussed above, a_{CB} and a_{CA} will each have two components as follows :

(*i*) a_{CB} has two components; a^r_{CB} and a^t_{CB} as shown by triangle $b'\,zc'$ in Fig. 8.2 (*b*), in which $b'\,z$ is parallel to BC and zc' is perpendicular to $b'\,z$ or BC.

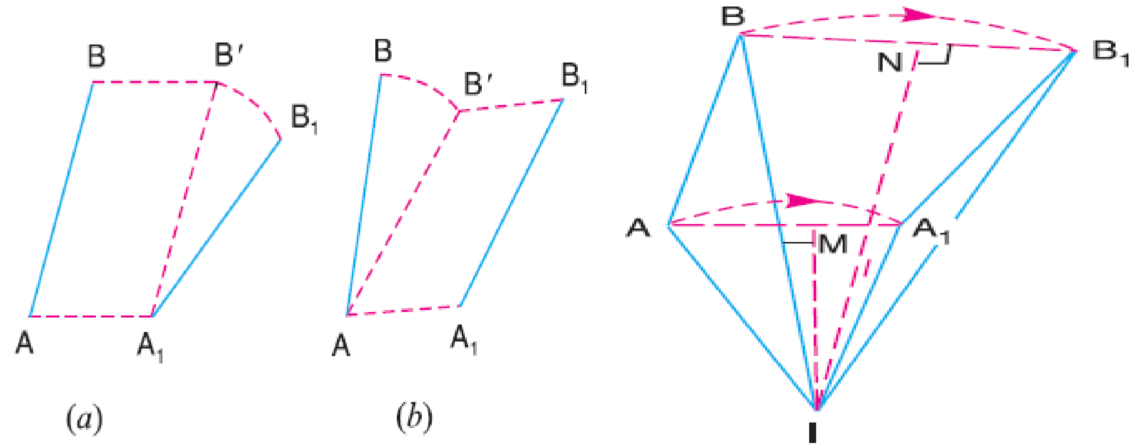
(*ii*) a_{CA} has two components ; a^r_{CA} and a^t_{CA} as shown by triangle $a'\,yc'$ in Fig. 8.2 (*b*), in which $a'\,y$ is parallel to $A\,C$ and yc' is perpendicular to $a'\,y$ or $A\,C$.

7. The angular acceleration of the link $A\,B$ is obtained by dividing the tangential components of the acceleration of B with respect to A (a^t_{BA}) to the length of the link. Mathematically, angular acceleration of the link $A\,B$,

$$\alpha_{AB} = a^t_{BA} / AB$$

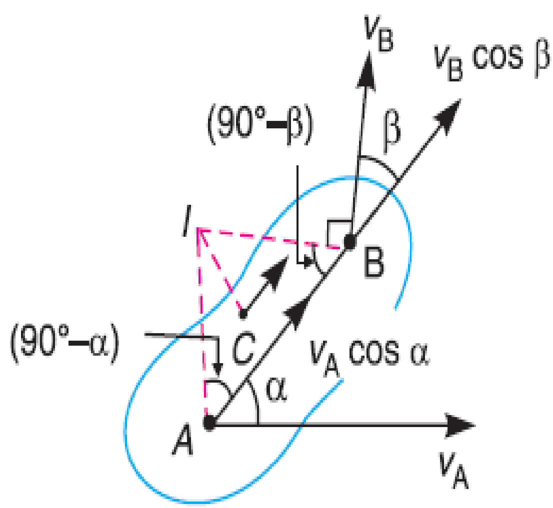
INSTANTANEOUS CENTER METHOD:

- Consider a rigid link AB, Which has two motions simultaneously.
- In fig (a), the link AB has first the motion of translation from AB to A_1B' and then rotation about A_1 till the final position of A_1 B_1.
- In fig (b), the link AB has first rotation about A from AB to AB' and translation from AB' to A_1 B_1 .
- In both the cases, the link neither has wholly rotation nor wholly translation but a combination of both the motions.
- The combined motion of link AB is assumed to be motion of pure rotation about a center known as "INSTANTANEOUS CENTER (I)".
- It is also known as Centro (or) Virtual Center.
- Locus of all the centros is called as CENTRODE.



Velocity of a Point on a Link by Instantaneous Centre Method:.

- Consider two points A and B on a rigid link.
- Let v_A and v_B are the velocities of points A and B with angles of α and β as shown in the fig.
- Draw AI and BI perpendicular to the directions of v_A and v_B respectively.
- These two intersect at I which is known as Instantaneous center.
- The complete link is to rotate about the center I.
- Since A and B are the points on a rigid link, therefore there cannot be a relative motion between A and B.



Now resolving the velocities along AB,

$$v_A \cos\alpha = v_B \cos\beta$$

or
$$\frac{v_A}{v_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90° - \beta)}{\sin(90° - \alpha)} \quad \text{...}(i)$$

Applying Lami's theorem to triangle ABI,

$$\frac{AI}{\sin(90° - \beta)} = \frac{BI}{\sin(90° - \alpha)}$$

or
$$\frac{AI}{BI} = \frac{\sin(90° - \beta)}{\sin(90° - \alpha)} \quad \text{...}(ii)$$

From equation (*i*) and (*ii*),

$$\frac{v_A}{v_B} = \frac{AI}{BI} \quad \text{or} \quad \frac{v_A}{AI} = \frac{v_B}{BI} = \omega \quad \text{...}(iii)$$

where ω = Angular velocity of the rigid link.

If C is any other point on the link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} \quad \text{...}(iv)$$

From the above equation, we see that

1. If v_A is known in magnitude and direction and v_B in direction only, then velocity of point B or any other point C lying on the same link may be determined in magnitude and direction.

2. The magnitude of velocities of the points on a rigid link is inversely proportional to the distances from the points to the instantaneous centre and is perpendicular to the line joining the point to the instantaneous centre.

Number of Instantaneous Centers in a Mechanism

The number of instantaneous centers in a constrained kinematic chain is equal to the number of possible combinations of two links. The number of pairs of links or the number of instantaneous centers is the number of combinations of n links taken two at a time. Mathematically, number of instantaneous centers,

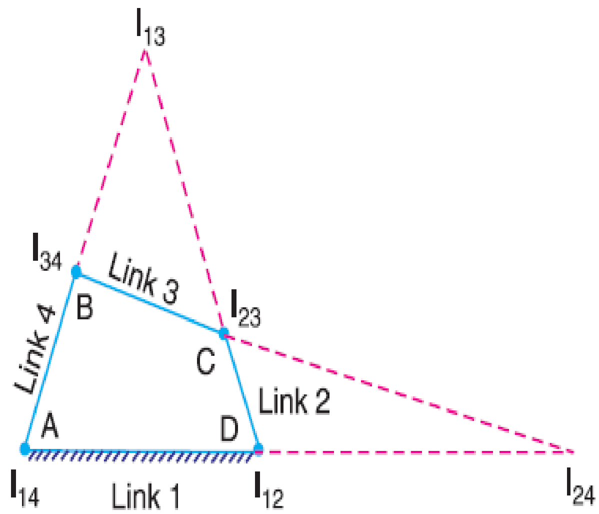
$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links.}$$

Types of Instantaneous Centres

- **Fixed instantaneous centers**
- **Permanent** instantaneous centers
- **Neither fixed nor permanent** instantaneous centers.

The first two are called as primary instantaneous centers and the third one is called as secondary instantaneous center.

Consider a four bar mechanism *ABCD as* shown in Fig. 5. The number of instantaneous centers (*N) in a four bar mechanism is given by*



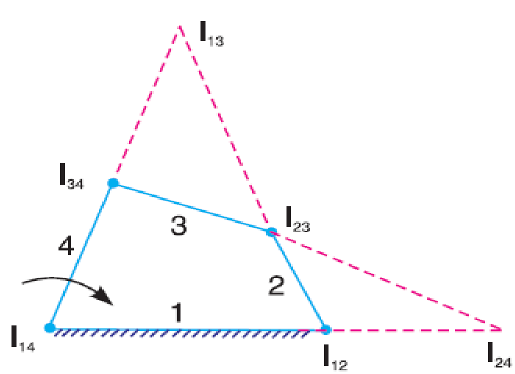
$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

- The instantaneous centers I_{12} *and* I_{14} *are called the* ***fixed instantaneous centers as they remain*** in the same place for all configurations of the mechanism.
- The instantaneous centers I_{23} *and* I_{34} are the ***permanent instantaneous centers as they move when the mechanism moves, but the joints*** are of permanent nature.
- The instantaneous centers I_{13} *and* I_{24} *are* ***neither fixed nor permanent instantaneous centers as they vary with the configuration of the mechanism.***

Method of Locating Instantaneous Centers in a Mechanism

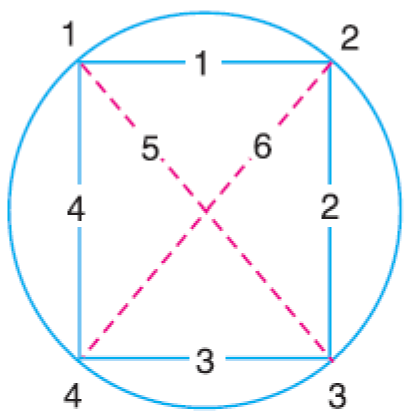
Consider a pin jointed four bar mechanism as shown in Fig. 8 (*a*). *The following procedure* is adopted for locating instantaneous centers.

- First of all, determine the number of instantaneous centers (N). *In this case* $N=6$.
- Make a list of all the instantaneous centers in a mechanism. Since for a four bar mechanism, there are six instantaneous centers, therefore these centers are listed as shown in the following table (known as book-keeping table)

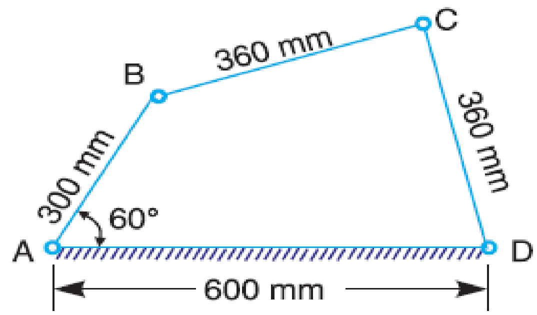


Links	1	2	3	4
Instantaneous centres (6 in number)	12 13 14	23 24	34	–

- Locate the fixed and permanent instantaneous centers by inspection. In the above Fig., I_{12} and I_{14} *are fixed instantaneous centers and* I_{23} *and* I_{34} *are permanent instantaneous centers.*
- Locate the remaining neither fixed nor permanent instantaneous centers (or secondary centers). This is done by circle diagram as shown in below Fig. *Mark points* on a circle equal to the number of links in a mechanism. In the present case, mark 1, 2, 3, and 4 on the circle.
- Join the points by solid lines to show that these centers are already found. In the circle diagram [Fig. 6.8 (*b*)] *these lines are 12, 23, 34 and 14 to indicate the centers* I_{12}, I_{23}, I_{34} *and* I_{14}.
- In order to find the other two instantaneous centers, join two such points that the line joining them forms two adjacent triangles in the circle diagram. The line which is responsible for completing two triangles, should be a common side to the two triangles. In Fig. *join 1 and 3* to form the triangles 123 and 341 and the instantaneous centre I_{13} *will lie on the intersection of* I_{12} I_{23} *and* I_{14} I_{34}*, produced if necessary, on the mechanism. Thus the instantaneous centre* I_{13} *is located.* Join 1 and 3 by a dotted line on the circle diagram and mark number 5 on it. Similarly the instantaneous centre I_{24} *will lie on the intersection of* I_{12} I_{14} *and* I_{23} I_{34}*, produced if necessary, on the mechanism.* Thus I_{24} *is located. Join 2 and 4 by a dotted line on the circle diagram and mark 6 on it. Hence* all the six instantaneous centers are located.

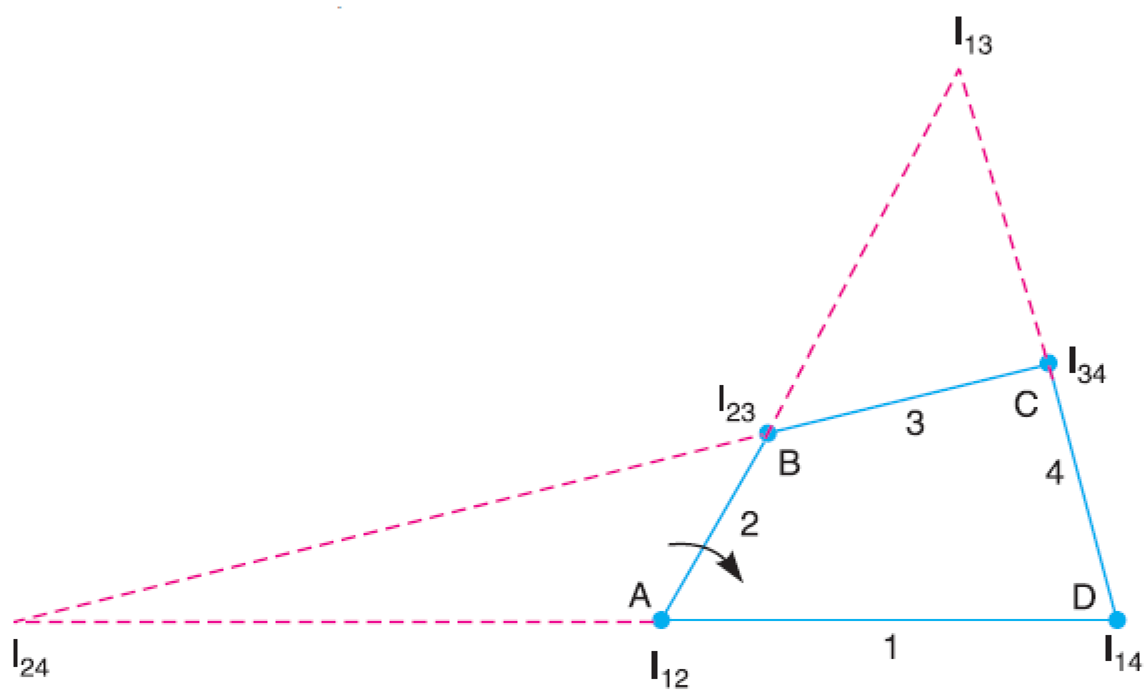


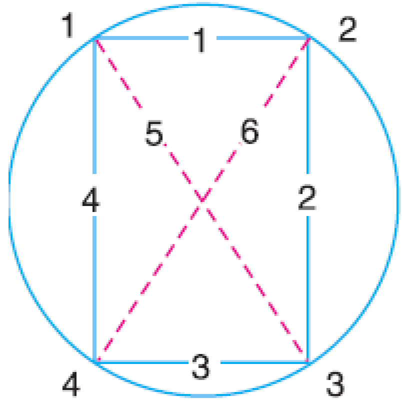
Problem: In a pin jointed four bar mechanism, as shown in Fig., AB=300mm, BC=CD=360mm, and AD=600mm. The angle BAD=60°. The crank AB rotates uniformly at 100r.p.m. Locate all the instantaneous centers and find the angular velocity of the link BC.



Solution. Given : $N_{AB} = 100$ *r.p.m or*
$\omega_{AB} = 2\pi \times 100/60 = 10.47$ rad/s
Since the length of crank $AB = 300$ *mm* $= 0.3$ *m,*
therefore velocity of point *B on link AB,*
$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141$ *m/s*
Location of instantaneous centers:

1. Since the mechanism consists of four links (*i.e. n = 4), therefore number of instantaneous* centers is 6.
2. For a four bar mechanism, the book keeping table may be drawn as discussed.
3. Locate the fixed and permanent instantaneous centers by inspection. These centers are I_{12}, I_{23}, I_{34} *and* I_{14}, *as shown in Fig.*
4. Locate the remaining neither fixed nor permanent instantaneous centers. This is done by circle diagram as shown in Fig. 6.11. Mark four points (equal to the number of links in a mechanism) 1, 2, 3, and 4 on the circle.





5. Join points 1 to 2, 2 to 3, 3 to 4 and 4 to 1 to indicate the instantaneous centers already located *i.e.* I_{12}, I_{23}, I_{34} *and* I_{14}.
6. Join 1 to 3 to form two triangles 1 2 3 and 3 4 1. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{13} *lies on the intersection of the lines joining the points* I_{12} I_{23} *and* I_{34} I_{14} *as shown in Fig. Thus centre* I_{13} *is located. Mark* number 5 (because four instantaneous centers have already been located) on the dotted line 1 3.
7. Now join 2 to 4 to complete two triangles 2 3 4 and 1 2 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore centre *I24 lies on the intersection of the lines* joining the points I_{23} *I34 and* I_{12} I_{14} *as shown in Fig. Thus centre* I_{24} is located. Mark number 6 on the dotted line 2 4. Thus all the six instantaneous centers are located.

Angular velocity of the link BC

Let ω_{BC} = Angular velocity of the link *BC*.

$$v_B = \omega_{BC} \times I_{13}\,B$$

By measurement, we find that $I_{13}\,B = 500\text{ mm} = 0.5\text{ m}$

$$\omega_{BC} = \frac{v_B}{I_{13}\,B} = \frac{3.141}{0.5} = 6.282\text{ rad/s} \quad \textbf{Ans.}$$

LOCATION OF INSTANTANEOUS CENTER

The following rules may be used in locating the instantaneous centres in a mechanism :

1. When the two links are connected by a pin joint (or pivot joint), the instantaneous centre lies on the centre of the pin as shown in Fig. (*a*). *Such a instantaneous centre is of permanent* nature, but if one of the links is fixed, the instantaneous centre will be of fixed type.

2. When the two links have a pure rolling contact (*i.e. link 2 rolls without slipping upon the* fixed link 1 which may be straight or curved), the instantaneous centre lies on their point of contact, as shown in Fig (*b*). *The velocity of any point A on the link 2 relative to fixed link 1 will be* perpendicular to I_{12} *A and is proportional to* I_{12} *A . In other words*

$$v_{A}\,/\,v_B = I_{12}\,A\,/\,I_{12}\,B$$

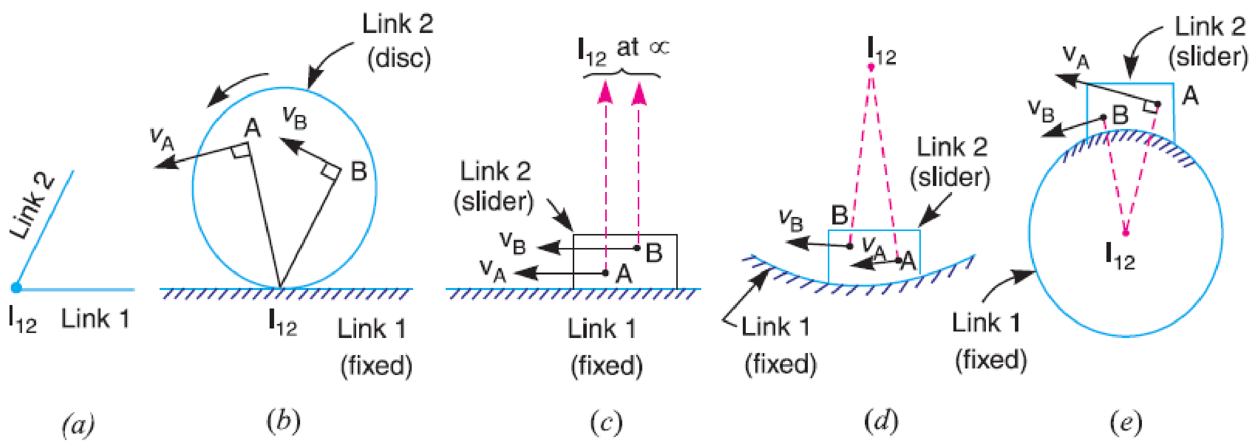
3. When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact. We shall consider the following three cases:

(a) When the link 2 (slider) moves on fixed link 1 having straight surface as shown in Fig. (*c*), *the instantaneous centre lies at infinity and each point on the slider have* the same velocity.

(b) When the link 2 (slider) moves on fixed link 1 having curved surface as shown in Fig.

(*d*) *The instantaneous centre lies on the centre of curvature of the curvilinear path* in the configuration at that instant.

(c) When the link 2 (slider) moves on fixed link 1 having constant radius of curvature as shown in Fig. (*e*), *the instantaneous centre lies at the centre of curvature i.e. the* centre of the circle, for all configuration of the links.



UNIT III
GYROSCOPIC COUPLE AND PRECESSIONAL MOTION

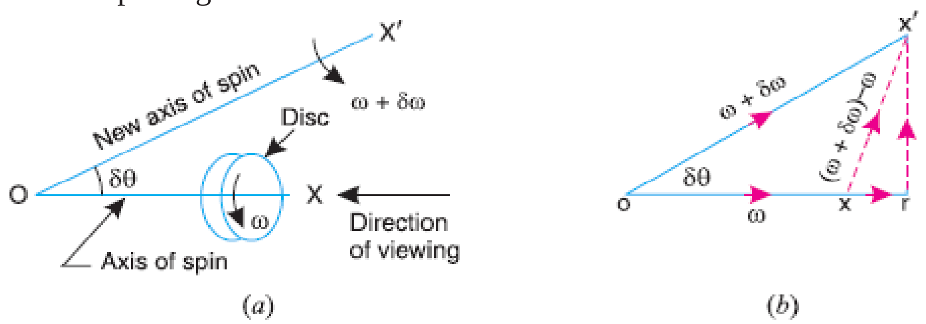
Introduction

1. When a body moves along a curved path with a uniform linear velocity, a force in the direction of centripetal acceleration (known as centripetal force) has to be applied externally over the body, so that it moves along the required curved path. This external force applied is known as **active force.**

2. When a body, itself, is moving with uniform linear velocity along a circular path, it is subjected to the centrifugal force* radially outwards. This centrifugal force is called **reactive force.** The action of the reactive or centrifugal force is to tilt or move the body along radially outward direction.

Precessional Angular Motion

We have already discussed that the angular acceleration is the rate of change of angular velocity with respect to time. It is a vector quantity and may be represented by drawing a vector diagram with the help of right hand screw rule.



Consider a disc, as shown in Fig (*a*), revolving or spinning about the axis *OX* (known as **axis of spin**) in anticlockwise when seen from the front, with an angular velocity ω in a plane at right angles to the paper.

After a short interval of time δ*t*, let the disc be spinning about the new axis of spin *OX* '(at an angle δθ) with an angular velocity (ω+ δω). Using the right hand screw rule, initial angular velocity of the disc (ω) is represented by vector *ox*; and the final angular velocity of the disc (ω+ δω) is represented by vector *ox*′ as shown in Fig. 14.1 (*b*). The vector *xx*′ represents the change of angular velocity in time δ*t i.e.* the angular acceleration of the disc. This may be resolved into two components, one parallel to *ox* and the other perpendicular to *ox*.

Component of angular acceleration in the direction of *ox*,

$$\alpha_t = \frac{xr}{\delta t} = \frac{or - ox}{\delta t} = \frac{ox' \cos \delta\theta - ox}{\delta t}$$

$$= \frac{(\omega + \delta\omega)\ \cos\delta\theta - \omega}{\delta t} = \frac{\omega\cos\delta\theta + \delta\omega\cos\delta\theta - \omega}{\delta t}$$

Since δθ is very small, therefore substituting cos δθ = 1, we have

$$\alpha_t = \frac{\omega + \delta\omega - \omega}{\delta t} = \frac{\delta\omega}{\delta t}$$

In the limit, when $\delta t \to 0$,

$$\alpha_t = \underset{\delta t \to 0}{\text{Lt}} \left(\frac{\delta\omega}{\delta t} \right) = \frac{d\omega}{dt}$$

Component of angular acceleration in the direction perpendicular to *ox*,

$$\alpha_c = \frac{rx'}{\delta t} = \frac{ox' \sin \delta\theta}{\delta t} = \frac{(\omega + \delta\omega) \sin \delta\theta}{\delta t} = \frac{\omega \sin \delta\theta + \delta\omega . \sin \delta\theta}{\delta t}$$

Since $\delta\theta$ in very small, therefore substituting $\sin \delta\theta = \delta\theta$, we have

$$\alpha_c = \frac{\omega . \delta\theta + \delta\omega . \delta\theta}{\delta t} = \frac{\omega . \delta\theta}{\delta t}$$

...(Neglecting $\delta\omega.\delta\theta$, being very small)

In the limit when $\delta t \to 0$,

$$\alpha_c = \underset{\delta t \to 0}{\text{Lt}} \frac{\omega . \delta\theta}{\delta t} = \omega \times \frac{d\theta}{dt} = \omega . \omega_P \qquad \ldots \left(\text{Substituting } \frac{d\theta}{dt} = \omega_P \right)$$

$\therefore$ Total angular acceleration of the disc

$$= \text{vector } xx' = \text{vector sum of } \alpha_t \text{ and } \alpha_c$$

$$= \frac{d\omega}{dt} + \omega \times \frac{d\theta}{dt} = \frac{d\omega}{dt} + \omega . \omega_P$$

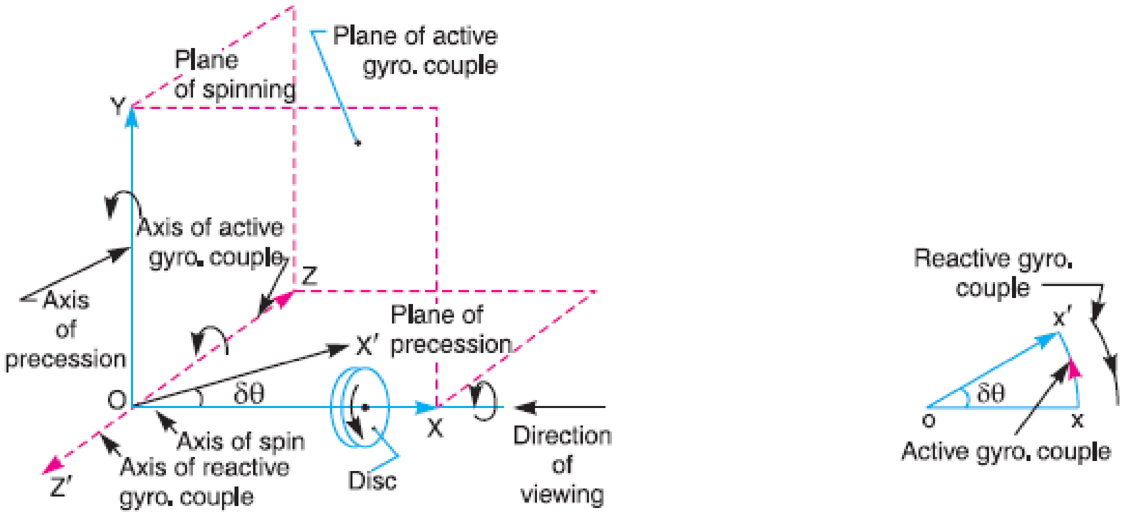
where dθ/dt is the angular velocity of the axis of spin about a certain axis, which is perpendicular to the plane in which the axis of spin is going to rotate. This angular velocity of the axis of spin (i.e.dθ/dt) is known as angular velocity of precession and is denoted by ω_P. The axis, about which the axis of spin is to turn, is known as axis of precession. The angular motion of the axis of spin about the axis of precession is known as precessional angular motion.

Gyroscopic Couple

Consider a disc spinning with an angular velocity ω rad/s about the axis of spin OX, in anticlockwise direction when seen from the front, as shown in Fig. 14.2 (a). Since the plane in which the disc is rotating is parallel to the plane YOZ, therefore it is called plane of spinning. The plane XOZ is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY (which is perpendicular to both the axes OX and OZ) at an angular velocity _P rad/s. This horizontal plane XOZ is called plane of precession and OY is the axis of precession.

Let I = Mass moment of inertia of the disc about OX, and
ω= Angular velocity of the disc.
Angular momentum of the disc = $I.\ \omega$

Since the angular momentum is a vector quantity, therefore it may be represented by the vector *ox*′ , as shown in Fig. 14.2 (*b*). The axis of spin *OX* is also rotating anticlockwise when seen from the top about the axis *OY*. Let the axis *OX* is turned in the plane *XOZ* through a small angle δθ radians to the position *OX*′, in time δ*t* seconds. Assuming the angular velocity ω to be constant, the angular momentum will now be represented by vector *ox*′.



(*a*) (*b*)

Change in angular momentum

$= \vec{ox'} - \vec{ox} = \vec{xx'} = \vec{ox}.\delta\theta$ ...(in the direction of $\vec{xx'}$)

$= I.\ \omega.\delta\theta$

and rate of change of angular momentum

$$= I.\omega \times \frac{\delta\theta}{dt}$$

Since the rate of change of angular momentum will result by the application of a couple to the disc, therefore the couple applied to the disc causing precession,

$$C = \underset{\delta t \to 0}{\mathrm{Lt}}\ I.\omega \times \frac{\delta\theta}{\delta t} = I.\omega \times \frac{d\theta}{dt} = I.\omega.\omega_P \qquad \ldots\left(\because \frac{d\theta}{dt} = \omega_P\right)$$

where ω_P = Angular velocity of precession of the axis of spin or the speed of rotation of the axis of spin about the axis of precession *OY.*

1. The couple $I.\omega.\omega_p$, in the direction of the vector *xx′* (representing the change in angular momentum) is the *active gyroscopic couple*, which has to be applied over the disc when the axis of spin is made to rotate with angular velocity ω_P about the axis of precession. The vector *xx′* lies in the plane *XOZ* or the horizontal plane. In case of a very small displacement δθ, the vector *xx′* will be perpendicular to the vertical plane *XOY*. Therefore the couple causing this change in the angular momentum will lie in the plane *XOY*. The vector *xx′*, as shown in Fig(*b*), represents an anticlockwise couple in the plane *XOY*. Therefore, the plane *XOY* is called the *plane of active gyroscopic couple* and the axis *OZ* perpendicular to the plane *XOY*, about which the couple acts, is called the axis of active gyroscopic couple.

2. When the axis of spin itself moves with angular velocity ω_P, the disc is subjected to *reactive couple* whose magnitude is same (*i.e.* $I.\omega.\omega_P$) but opposite in direction to that of active couple. This reactive couple to which the disc is subjected when the axis of spin rotates about the axis of precession is known as *reactive gyroscopic couple*. The axis of the reactive gyroscopic couple is represented by *OZ′* in Fig(*a*).

3. The gyroscopic couple is usually applied through the bearings which support the shaft. The bearings will resist equal and opposite couple.
4. The gyroscopic principle is used in an instrument or toy known as gyroscope. The gyroscopes are installed in ships in order to minimize the rolling and pitching effects of waves. They are also used in Aeroplanes, monorail cars, gyrocompasses etc.

Effect of the Gyroscopic Couple on an Aeroplane

The top and front view of an aeroplane are shown in Fig (*a*). Let engine or propeller rotates in the clockwise direction when seen from the rear or tail end and the aeroplane takes a turn to the left.

Let ω= Angular velocity of the engine in rad/s,

m = Mass of the engine and the propeller in kg,

k = Its radius of gyration in metre

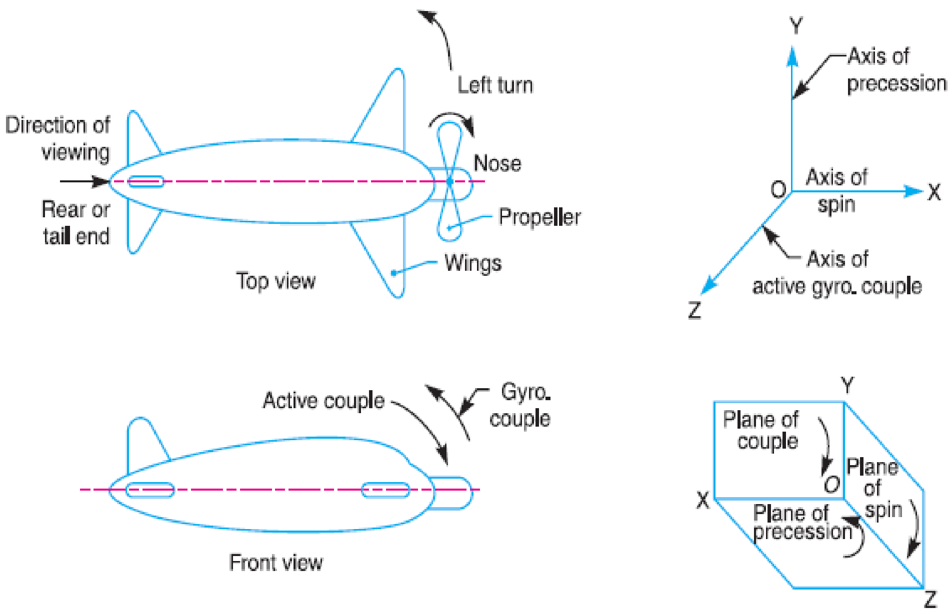
I = Mass moment of inertia of the engine and the propeller in kg-m^2 $= m.k^2$,

v = Linear velocity of the aeroplane in m/s,

R = Radius of curvature in metres, and

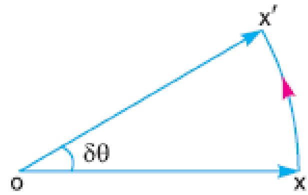
ω_P = Angular velocity of precession=V/R

Gyroscopic couple acting on the aeroplane, $C = I.\omega.\omega P$

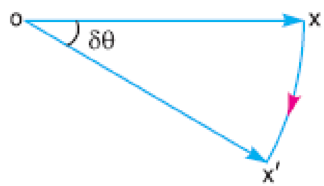


Before taking the left turn, the angular momentum vector is represented by *ox*. When it takes left turn, the active gyroscopic couple will change the direction of the angular momentum vector from *ox* to *ox′* as shown in Fig(*a*). The vector *xx′*,in the limit, represents the change of angular momentum or the active gyroscopic couple and is perpendicular to *ox*. Thus the plane of active gyroscopic couple *XOY* will be perpendicular to *xx′*, *i.e.* vertical in this case, as shown in Fig (*b*). By applying right hand screw rule to vector *xx′*, we find that the direction of active gyroscopic couple is clockwise as shown in the front view of Fig (*a*).

In other words, for left hand turning, the active gyroscopic couple on the aeroplane in the axis *OZ* will be clockwise as shown in Fig (*b*).The reactive gyroscopic couple (equal in magnitude of active gyroscopic couple) will act in the opposite direction (*i.e.* in the anticlockwise direction) and the effect of this couple is, therefore, to raise the nose and dip the tail of the aeroplane.



(*a*) Aeroplane taking left turn.

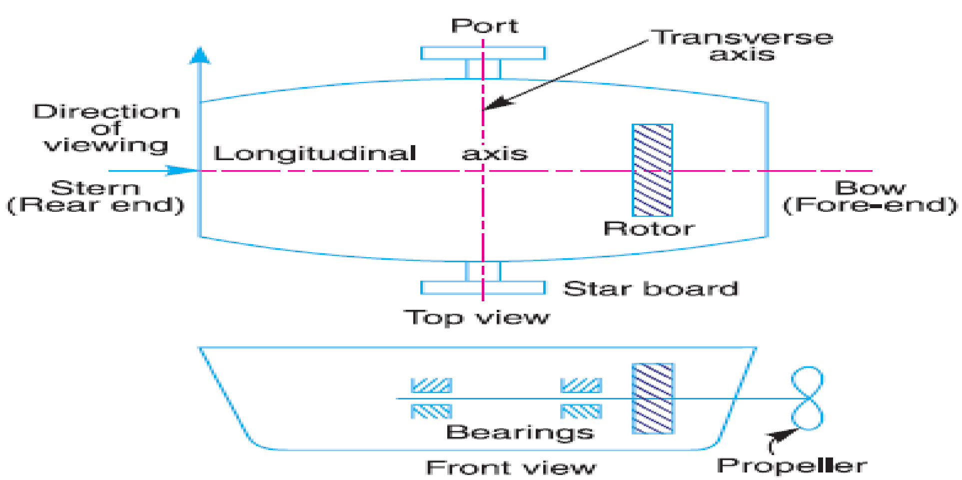


(*b*) Aeroplane taking right turn.

Terms Used in a Naval Ship

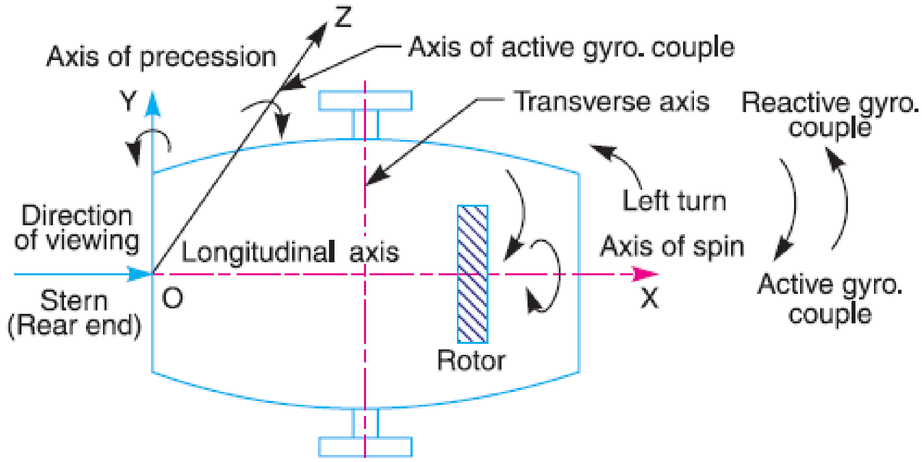
The top and front views of a naval ship are shown in Fig 14.7. The fore end of the ship is called ***bow*** and the rear end is known as ***stern*** or ***aft***. The left hand and right hand sides of the ship, when viewed from the stern are called ***port*** and ***star-board*** respectively. We shall now discuss the effect of gyroscopic couple on the naval ship in the following three cases:

1. Steering,
2. Pitching,
3. Rolling.

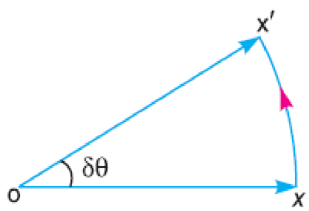


Effect of Gyroscopic Couple on a Naval Ship during Steering

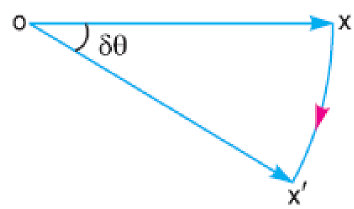
Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Fig. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aeroplane.



When the rotor of the ship rotates in the clockwise direction when viewed from the stern, it will have its angular momentum vector in the direction ox as shown in Fig(a). As the ship steers to the left, the active gyroscopic couple will change the angular momentum vector from ox to ox′. The vector xx′ now represents the active gyroscopic couple and is perpendicular to ox. Thus the plane of active gyroscopic couple is perpendicular to xx′ and its direction in the axis OZ for left hand turn is clockwise as shown in Fig. The reactive gyroscopic couple of the same magnitude will act in the opposite direction (i.e. in anticlockwise direction). The effect of this reactive gyroscopic couple is to raise the bow and lower the stern.



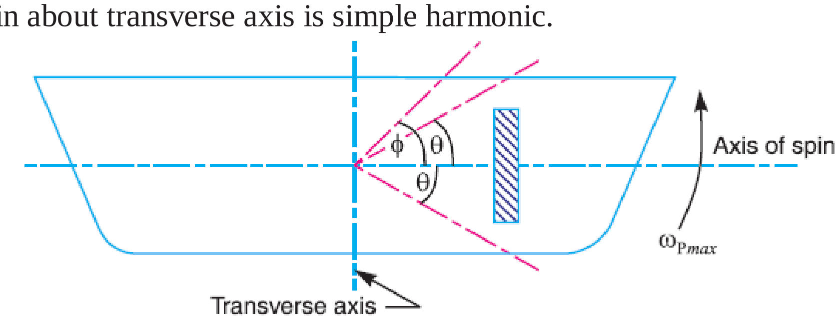
(*a*) Streeing to the left



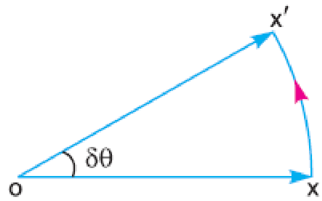
(*b*) Streeing to the right

Effect of Gyroscopic Couple on a Naval Ship during Pitching

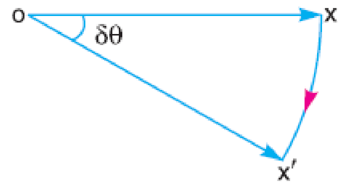
Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis, as shown in Fig(*a*). In this case, the transverse axis is the axis of precession. The pitching of the ship is assumed to take place with simple harmonic motion *i.e.* the motion of the axis of spin about transverse axis is simple harmonic.



(*a*) Pitching of a naval ship



(*b*) Pitching upward



(*c*) Pitching downward

∴ Angular displacement of the axis of spin from mean position after time t seconds,

$$\theta = \phi \sin \omega_1 . t$$

ϕ = Amplitude of swing *i.e.* maximum angle turned from the mean position in radians, and

ω_1 = Angular velocity of S.H.M.

$$= \frac{2\pi}{\text{Time period of S.H.M. in seconds}} = \frac{2\pi}{t_p} \text{ rad/s}$$

Angular velocity of precession,

$$\omega_P = \frac{d\theta}{dt} = \frac{d}{dt}(\phi \sin \omega_1 . t) = \phi\, \omega_1 \cos \omega_1 t$$

The angular velocity of precession will be maximum, if $\cos \omega_1 . t = 1$.

∴ Maximum angular velocity of precession,

$$\omega_{Pmax} = \phi . \omega_1 = \phi \times 2\pi / t_p \qquad \text{...(Substituting } \cos \omega_1 . t = 1)$$

Let I = Moment of inertia of the rotor in kg-m^2, and
ω= Angular velocity of the rotor in rad/s.
Mamimum gyroscopic couple,

$$C_{max} = I.\ \omega.\ \omega_{Pmax}$$

When the pitching is upward, the effect of the reactive gyroscopic couple, as shown in Fig.(*b*), will try to move the ship toward star-board. On the other hand, if the pitching is downward, the effect of the reactive gyroscopic couple, as shown in Fig(*c*), is to turn the ship towards port side.

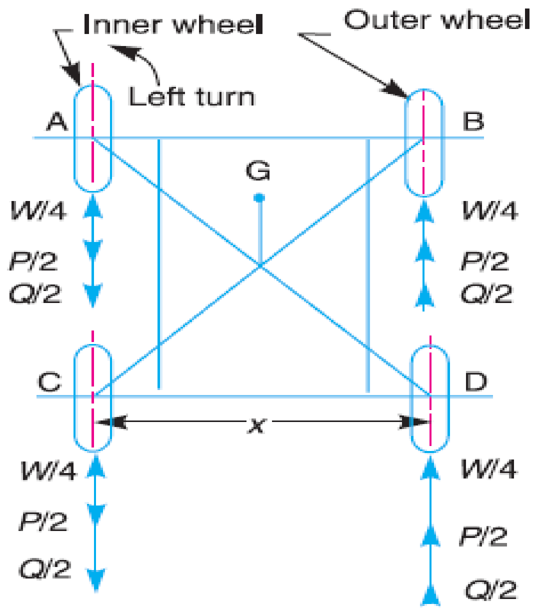
Effect of Gyroscopic Couple on a Naval Ship during Rolling

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship.

In case of rolling of a ship, the axis of precession (*i.e.* longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

Stability of a Four Wheel Drive Moving in a Curved Path

Consider the four wheels A, B, C and D of an automobile locomotive taking a turn towards left as shown in Fig. The wheels A and C are inner wheels, whereas B and D are outer wheels. The centre of gravity ($C.G.$) of the vehicle lies vertically above the road surface.



Let m = Mass of the vehicle in kg,

W = Weight of the vehicle in newtons = $m.g$,

r_W = Radius of the wheels in metres,

R = Radius of curvature in metres ($R > r_W$),

h = Distance of centre of gravity, vertically above the road surface in metres,

x = Width of track in metres,

I_W = Mass moment of inertia of one of the wheels in kg-m^2,

ω_W = Angular velocity of the wheels or velocity of spin in rad/s,

I_E = Mass moment of inertia of the rotating parts of the engine in kg-m2,

ω_E = Angular velocity of the rotating parts of the engine in rad/s,

G = Gear ratio $=\omega_E/\omega_w$

v = Linear velocity of the vehicle in m/s = $\omega_W.r_W$

A little consideration will show, that the weight of the vehicle (W) will be equally distributed over the four wheels which will act downwards. The reaction between each wheel and the road surface of the same magnitude will act upwards.

Therefore

Road reaction over each wheel

$= W/4 = m.g/4$ newtons

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle.

1. **Effect of the gyroscopic couple**

Since the vehicle takes a turn towards left due to the precession and other rotating parts, therefore a gyroscopic couple will act.
We know that velocity of precession,
$\omega_P = v/R$
Gyroscopic couple due to 4 wheels,
$C_W = 4\, I_W.\omega_W.\omega_P$
and gyroscopic couple due to the rotating parts of the engine,
$C_E = I_E.\omega_E.\omega_P = I_E.G.\omega_W.\omega_P$
Net gyroscopic couple,
$C = C_W \pm C_E = 4\, I_W.\omega_W.\omega_P \pm I_E.G.\omega_W.\omega_P$
$= \omega_W.\omega_P\, (4\, I_W \pm G.I_E)$

The ***positive*** sign is used when the wheels and rotating parts of the engine rotate in the same direction. If the rotating parts of the engine revolves in opposite direction, then ***negative*** sign is used.

Due to the gyroscopic couple, vertical reaction on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels.
Let the magnitude of this reaction at the two outer or inner wheels be P newtons. Then

$$P \times x = C \text{ or } P = C/x$$

Vertical reaction at each of the outer or inner wheels,

$$P/2 = C/2x$$

2. **Effect of the centrifugal couple**

Since the vehicle moves along a curved path, therefore centrifugal force will act outwardly at the centre of gravity of the vehicle. The effect of this centrifugal force is also to overturn the vehicle.
We know that centrifugal force,

$$F_C = \frac{m \times v^2}{R}$$

The couple tending to overturn the vehicle or overturning couple,

$$C_O = F_C \times h = \frac{m.v^2}{R} \times h$$

This overturning couple is balanced by vertical reactions, which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be Q. Then

$$Q \times x = C_O \quad \text{or} \quad Q = \frac{C_O}{x} = \frac{m.v^2.h}{R.x}$$

Vertical reaction at each of the outer or inner wheels,

$$\frac{Q}{2} = \frac{m.v^2.h}{2R.x}$$

Total vertical reaction at each of the outer wheel,

$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

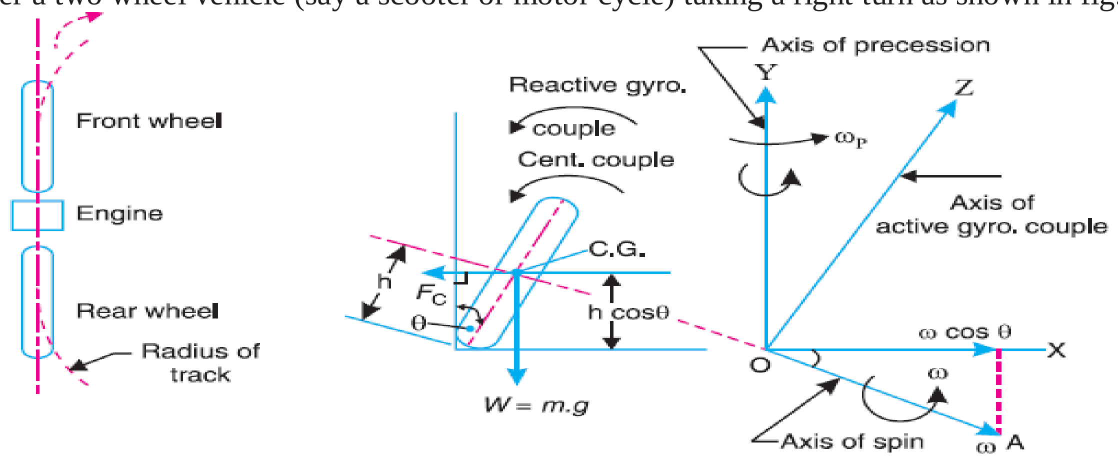
total vertical reaction at each of the inner wheel

$$P_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

A little consideration will show that when the vehicle is running at high speeds, P_I may be zero or even negative. This will cause the inner wheels to leave the ground thus tending to overturn the automobile. In order to have the contact between the inner wheels and the ground, the sum of $P/2$ and $Q/2$ must be less than $W/4$.

Stability of a Two Wheel Vehicle Taking a Turn

Consider a two wheel vehicle (say a scooter or motor cycle) taking a right turn as shown in fig.



(a) (b) (c)

Let m = Mass of the vehicle and its rider in kg,

W = Weight of the vehicle and its rider in newtons = $m.g$,

h = Height of the centre of gravity of the vehicle and rider,

r_W = Radius of the wheels,

R = Radius of track or curvature,

I_W = Mass moment of inertia of each wheel,

I_E = Mass moment of inertia of the rotating parts of the v engine,

ω_W = Angular velocity of the wheels,

ω_E = Angular velocity of the engine,

G = Gear ratio = ω_E / ω_W,

v = Linear velocity of the vehicle = $\omega_W \times r_W$,

θ = Angle of heel. It is inclination of the vehicle to the vertical for equilibrium.

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle,

1. **Effect of gyroscopic couple**

We know that $v = \omega_W \times r_W$ or $\omega_W = v / r_W$

$$\omega_E = G.\omega_W = G \times \frac{v}{r_W}$$

$\therefore$ Total $(I \times \omega) = 2\,I_W \times \omega_W \pm I_E \times \omega_E$

$$= 2\,I_W \times \frac{v}{r_W} \pm I_E \times G \times \frac{v}{r_W} = \frac{v}{r_W}(2\,I_W \pm G.I_E)$$

velocity of precession, $\omega_P = v / R$

A little consideration will show that when the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane as shown in Fig(*b*). This angle is known as ***angle of heel.*** In other words, the axis of spin is inclined to the horizontal at an angle θ, as shown in Fig (*c*). Thus the angular momentum vector $I\omega$ due to spin is represented by *OA* inclined to *OX* at an angle θ. But the precession axis is vertical. Therefore the spin vector is resolved along *OX*.

Gyroscopic couple,

$$C_1 = I.\omega \cos\theta \times \omega_P = \frac{v}{r_W}(2\,I_W \pm G.I_E)\cos\theta \times \frac{v}{R}$$

$$= \frac{v^2}{R.r_W}(2\,I_W \pm G.I_E)\cos\theta$$

2. Effect of centrifugal couple

We know that centrifugal force,

$$F_C = \frac{m.v^2}{R}$$

This force acts horizontally through the centre of gravity (*C.G.*) along the outward direction.
Centrifugal couple,

$$C_2 = F_C \times h\cos\theta = \left(\frac{m.v^2}{R}\right) h\cos\theta$$

Since the centrifugal couple has a tendency to overturn the vehicle, therefore

Total overturning couple,

$$C_O = \text{Gyroscopic couple} + \text{Centrifugal couple}$$

$$= \frac{v^2}{R.r_W}(2\,I_W + G.I_E)\cos\theta + \frac{m.v^2}{R} \times h\cos\theta$$

$$= \frac{v^2}{R}\left[\frac{2\,I_W + G.I_E}{r_W} + m.h\right]\cos\theta$$

We know that balancing couple = $m.g.h \sin\theta$

The balancing couple acts in clockwise direction when seen from the front of the vehicle.

Therefore for stability, the overturning couple must be equal to the balancing couple, *i.e.*

$$\frac{v^2}{R}\left(\frac{2\,I_W + G.I_E}{r_W} + m.h\right)\cos\theta = m.g.h\sin\theta$$

From this expression, the value of the angle of heel (θ) may be determined, so that the vehicle does not skid.

PROBLEMS

Example 1. A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis?

Solution. Given: d = 300 mm or r = 150 mm = 0.15 m ; m = 5 kg ; l = 600 mm = 0.6 m ;N = 300 r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

We know that the mass moment of inertia of the disc, about an axis through its centre of gravity and perpendicular to the plane of disc,

$I = m.r^{2/}2 = 5(0.15)2/2 = 0.056$ kg-m^2

couple due to mass of disc,

$C = m.g.l = 5 \times 9.81 \times 0.6 = 29.43$ N-m

Let ω_P = Speed of precession.

We know that couple (C),

$$29.43 = I.\omega.\omega_P = 0.056 \times 31.42 \times \omega_P = 1.76\,\omega_P$$

$$\omega_P = 29.43/1.76 = 16.7 \text{ rad/s}$$

Example 2. An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 k gand a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.

Solution. Given : R = 50 m ; v = 200 km/hr = 55.6 m/s ; m = 400 kg ; k = 0.3 m ; N = 2400 r.p.m. or $\omega = 2\pi \times 2400/60 = 251$ rad/s

We know that mass moment of inertia of the engine and the propeller,
$I = m.k^2 = 400(0.3)2 = 36$ kg-m^2

angular velocity of precession,
$\omega_P = v/R = 55.6/50 = 1.11$ rad/s
We know that gyroscopic couple acting on the aircraft,
$C = I.\ \omega.\ \omega_P = 36 \times 251.4 \times 1.11 = 100\ 46$ N-m
10.046 kN-m
when the aeroplane turns towards left, the effect of the gyroscopic couple is to lift the nose upwards and tail downwards.

Example3. The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius.

Solution. Given: m = 8 t = 8000 kg ; k = 0.6 m ; N = 1800 r.p.m. or $\omega = 2\pi \times 1800/60 = 188.5$ rad/s ; v = 100 km/h = 27.8 m/s ; R = 75 m

We know that mass moment of inertia of the rotor,

$I = m.k^2 = 8000\ (0.6)2 = 2880$ kg-m^2

angular velocity of precession,

$\omega_P = v\ /\ R = 27.8 / 75 = 0.37$ rad/s

We know that gyroscopic couple,
$C = I.\omega.\omega_P = 2880 \times 188.5 \times 0.37 = 200\ 866$ N-m
= 200.866 kN-m

when the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

UNIT IV

CAMS AND FOLLOWERS

Introduction: A cam is a rotating machine element which gives reciprocating or oscillating motion to another element known as follower. The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at uniform speed by a shaft, but the follower motion is predetermined and will be according to the shape of the cam. The cam and follower is one of the simplest as well as one of the most important mechanisms found in modern machinery today. The cams are widely used for operating the inlet and exhaust valves of internal combustion engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathes etc.

Classification of Followers:

The followers may be classified as discussed below:

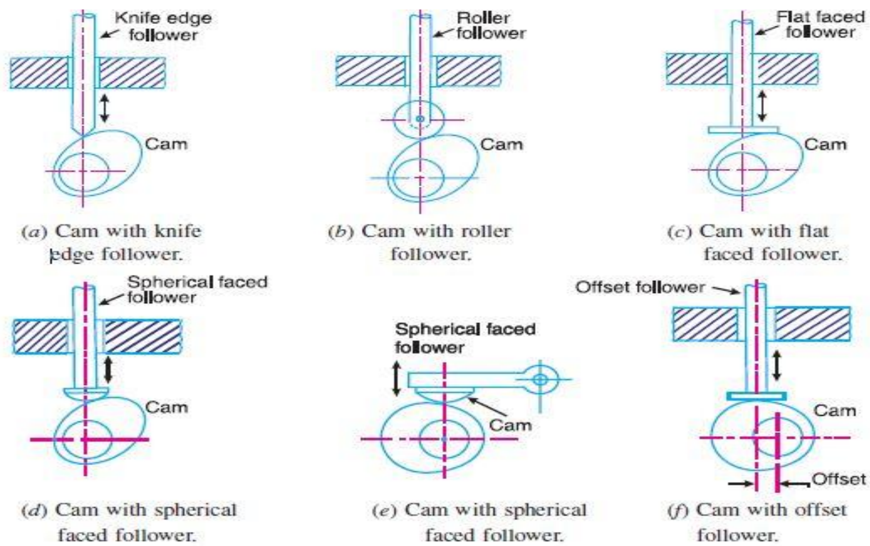
1. According to the surface in contact. The followers, according to the surface in contact, are as follows

(a)Knife edge follower. When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower, as shown in Fig. The sliding motion takes place between the contacting surfaces (i.e. the knife edge and the cam surface. It is seldom used in practice because the small area of contacting surface results in excessive wear. In knife edge followers, a considerable side thrust exists between the follower and the guide.

(b) Roller follower. When the contacting end of the follower is a roller, it is called a roller follower, as shown in Fig. 20.1 (b). Since the rolling motion takes place between the contacting surfaces (i.e. the roller and the cam), therefore the rate of wear is greatly reduced. In roller followers also the side thrust exists between the follower and the guide. The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircraft engines.

(c) Flat faced or mushroom follower. When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower, as shown in Fig. 20.1 (c). It may be noted that the side thrust between the follower and the guide is much reduced in case of flat faced followers. The only side thrust is due to friction between the contact surfaces of the follower and the cam. The relative motion between these surfaces is largely of sliding nature but wear may be reduced by off-setting the axis of the follower, as shown in Fig. 20.1 (f) so that when the cam rotates, the follower also rotates about its own axis. The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobile engines.

(d) Spherical faced follower. When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in Fig. 20.1 (d). It may be noted that when order to minimize these stresses, the flat end of the follower is machined to a spherical shape



(a) Cam with knife edge follower. (b) Cam with roller follower. (c) Cam with flat faced follower.

(d) Cam with spherical faced follower. (e) Cam with spherical faced follower. (f) Cam with offset follower.

2. According to the motion of the follower. The followers, according to its motion, are of the following two types:

(a) Reciprocating or translating follower. When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. The followers as shown in Fig. 20.1 (a) to (d) are all reciprocating or translating followers.

(b) Oscillating or rotating follower. When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower. The follower, as shown in Fig 20.1 (e), is an oscillating or rotating follower.

3. According to the path of motion of the follower. The followers, according to its path of motion, are of the following two types:

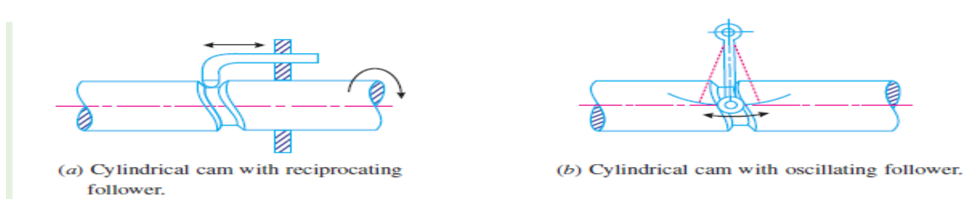
(a) Radial follower. When the motion of the follower is along an axis passing through the centre of the cam, it is known as radial follower. The followers, as shown in Fig. 20.1 (a)to (e), are all radial followers.

(b) Off-set follower. When the motion of the follower is along an axis away from the axis of the cam centre, it is called off-set follower. The follower, as shown in Fig. 20.1 (f), is an off-set follower.

Classification of Cams:

Though the cams may be classified in many ways, yet the following two types are important

From the subject point of view:



(a) Cylindrical cam with reciprocating follower. (b) Cylindrical cam with oscillating follower.

1. Radial or disc cam. In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis. The cams as shown in   are all radial cams.

2. Cylindrical cam. In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower is shown in respectively

Terms Used in Radial Cams

A radial cam with reciprocating roller follower is shown in Fig. The following terms are important in order to draw the cam profile.

1. Base circle. It is the smallest circle that can be drawn to the cam profile.

2. Trace point. It is a reference point on the follower and is used to generate the pitch curve. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centre of the roller represents the trace point.

3. Pressure angle. It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

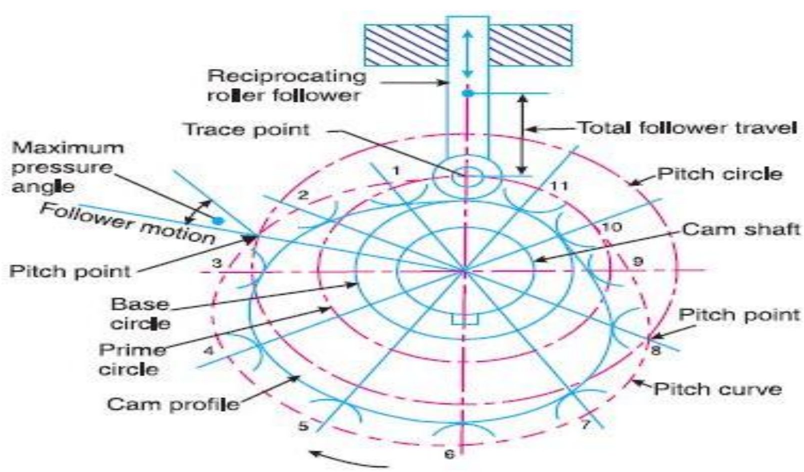
4. Pitch point. It is a point on the pitch curve having the maximum pressure angle.

5. Pitch circle. It is a circle drawn from the centre of the cam through the pitch points.

6. Pitch curve. It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.

7. Prime circle. It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

8. Lift or stroke. It is the maximum travel of the follower from its lowest position to the top most position.

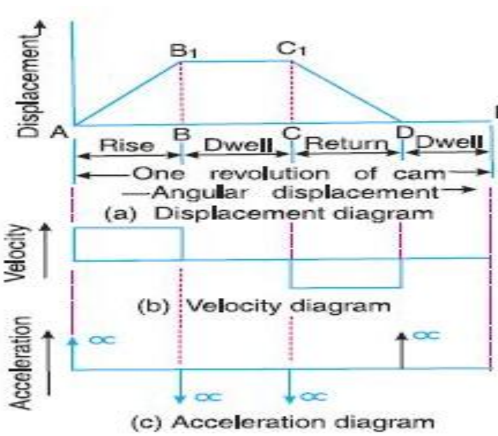


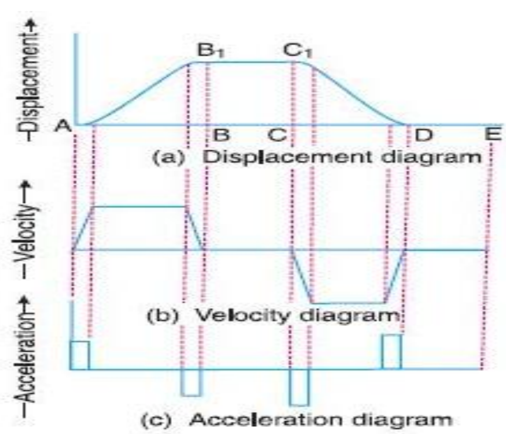
Motion of the Follower

1. **Uniform velocity**

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity

The displacement, velocity and acceleration diagrams when a knife-edged follower moves with uniform velocity are shown in respectively. The abscissa (base) represents the time (i.e. the number of seconds required for the cam to complete one revolution) or it may represent the angular displacement of the cam in degrees. The ordinate represents the displacement, or velocity or acceleration of the follower. Since the follower moves with uniform velocity during its rise and return stroke, therefore the slope of the displacement curves must be constant. In other words, AB_1 and C_1 D must be straight lines. A little consideration will show that the follower remains at rest during part of the cam rotation. The periods during which the follower remains at rest are known dwell periods, as shown by lines B C_1 and DE we see that the acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This is due to the fact that the follower is required to start from rest and has to gain a velocity within no time. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite. These conditions are however, impracticable.





Simple harmonic motion

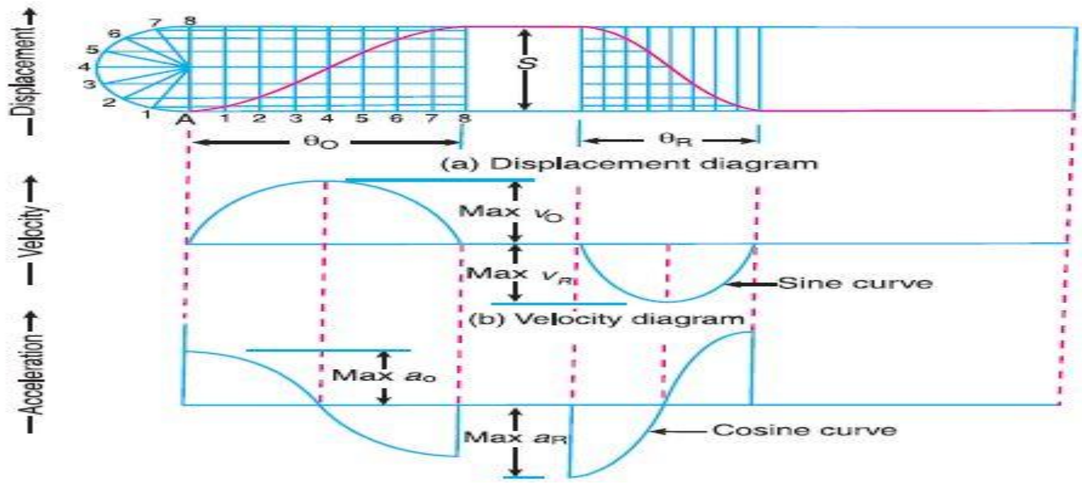
The displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion respectively. The displacement diagram is drawn as follows:

1. Draw a semi-circle on the follower stroke as diameter.

2. Divide the semi-circle into any number of even equal parts (say eight).

3. Divide the angular displacements of the cam during out stroke and return stroke into the same number of equal parts.

4. The displacement diagram is obtained by projecting the points as shown in Fig. 20.6 (a).The velocity and acceleration diagrams are shown in Fig. 20.6 (b) and (c) respectively. Since the follower moves with a simple harmonic motion, therefore velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve. We see from Fig. 20.6 (b) that the velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke. On the other hand, the acceleration of the follower is maximum at the beginning and at the ends of the stroke and diminishes to zero at mid-stroke.



Consider a point P moving at a uniform speed ω_P radians per sec round the circumference of a circle with the stroke S as diameter, as shown in Fig. 20.7. The point P' (which is the projection of a point P on the diameter) executes a simple harmonic motion as the point P rotates. The motion of the follower is similar to that of point P'.

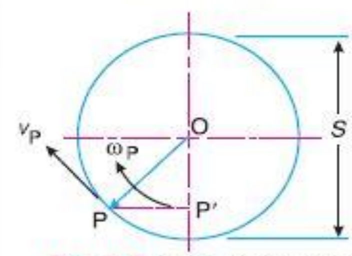


Fig. 20.7. Motion of a point.

$\therefore$ Peripheral speed of the point P',

$$v_P = \frac{\pi S}{2} \times \frac{1}{t_O} = \frac{\pi S}{2} \times \frac{\omega}{\theta_O}$$

and maximum velocity of the follower on the outstroke,

$$v_O = v_P = \frac{\pi S}{2} \times \frac{\omega}{\theta_O} = \frac{\pi \omega S}{2\theta_O}$$

We know that the centripetal acceleration of the point P,

$$a_P = \frac{(v_P)^2}{OP} = \left(\frac{\pi \omega S}{2\theta_O}\right)^2 \times \frac{2}{S} = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

$\therefore$ Maximum acceleration of the follower on the outstroke,

$$a_O = a_P = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi \omega . S}{2\theta_R}$$

and maximum acceleration of the follower on the return stroke,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2}$$

Uniform acceleration and retardation

The displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation respectively. We see that the displacement diagram consists of a parabolic curve and may be drawn as discussed below:

1. Divide the angular displacement of the cam during outstroke into any even number of equal parts (say eight) and draw vertical lines through these points as shown in Fig. 20.8 (a).

2. Divide the stroke of the follower (S) into the same number of equal even parts.

3. Join Aa to intersect the vertical line through point 1 at B. Similarly, obtain the other points C, D etc. Now join these points to obtain the parabolic curve for the out stroke of the follower.

4. In the similar way as discussed above, the displacement diagram for the follower during return stroke may be drawn. Since the acceleration and retardation are uniform, therefore the velocity varies directly with the time. The velocity diagram is shown in

Let S = Stroke of the follower,

θ_O and θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively, and

ω = Angular velocity of the cam.

We know that time required for the follower during outstroke,

$$t_O = \theta_O / \omega$$

and time required for the follower during return stroke,

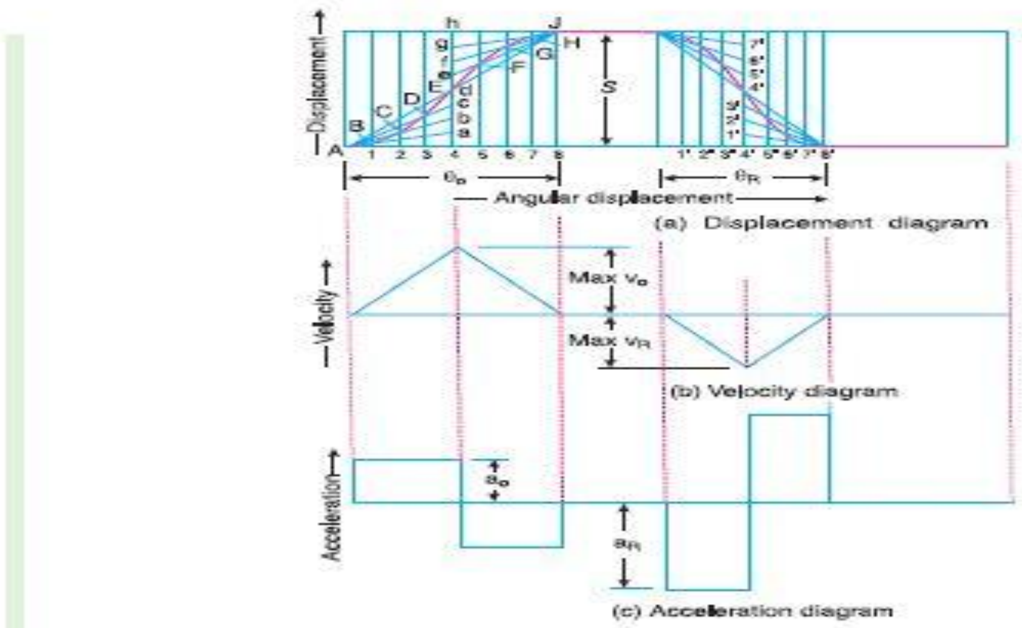
$$t_R = \theta_R / \omega$$

Mean velocity of the follower during outstroke

$$= S/t_O$$

and mean velocity of the follower during return stroke

$$= S/t_R$$



PROBLEM:

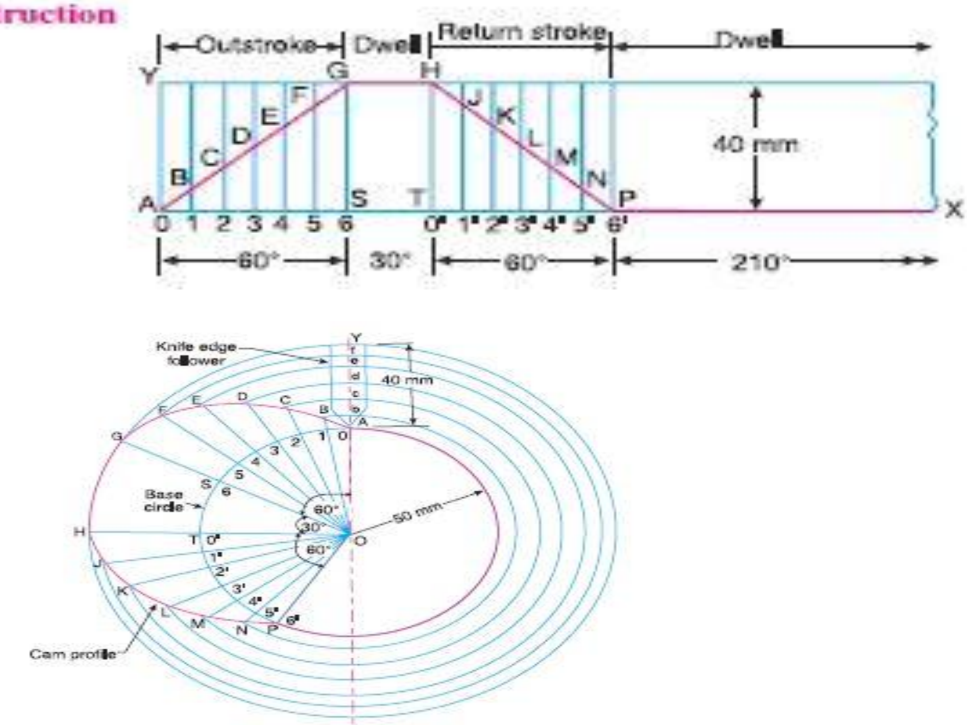
A cam is to give the following motion to a knife-edged follower :

1. Outstroke during 60° of cam rotation ;
2. Dwell for the next 30° of cam rotation ;
3. Return stroke during next 60° of cam rotation, and
4. Dwell for the remaining 210° of cam rotation.

The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes. Draw the profile of the cam when

(a) The axis of the follower passes through the axis of the cam shaft, and
(b) The axis of the follower is offset by 20 mm from the axis of the cam shaft.

Construction



A cam is to be designed for a knife edge follower with the following data :

1. Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion.
2. Dwell for the next 30°.
3. During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion.
4. Dwell during the remaining 180°.

Draw the profile of the cam when

(a) The line of stroke of the follower passes through the axis of the cam shaft, and
(b) The line of stroke is offset 20 mm from the axis of the cam shaft. The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

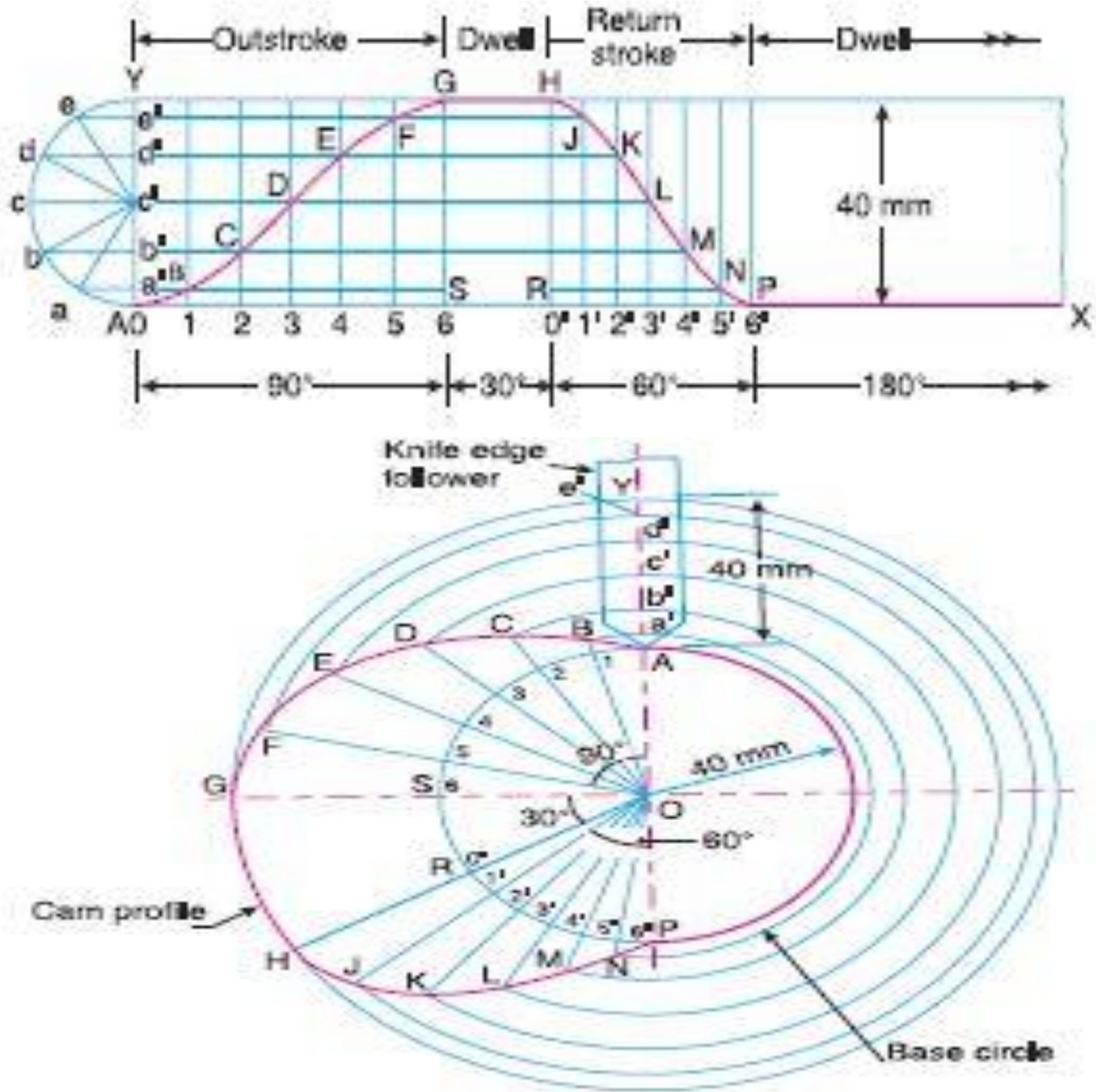

Outstroke
Dwell
Return stroke
Dwell
40 mm
90°
30°
60°
180°
Knife edge follower
40 mm
40 mm
90°
30°
60°
Cam profile
Base circle

Fig. 20.14

UNIT V

GEARS AND GEAR TRAINS

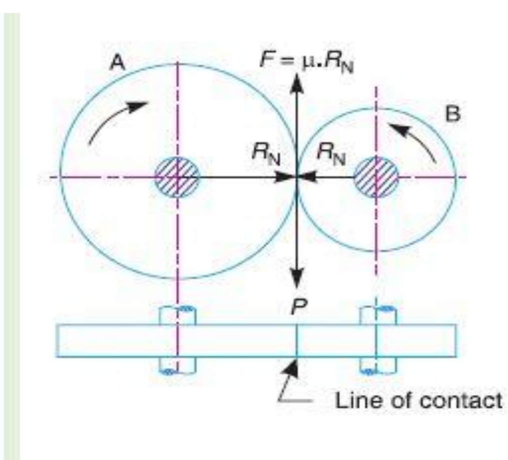
Introduction:

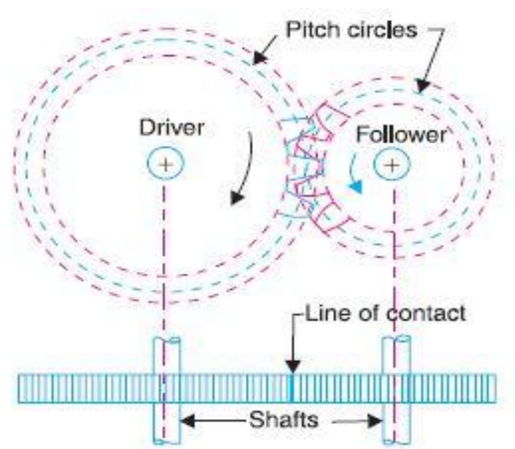
The slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by means of gears or toothed wheels. A gear drive is also provided, when the distance between the driver and the follower is very small.

Friction Wheels

The motion and power transmitted by gears is kinematically equivalent to that transmitted by friction wheels or discs. In order to understand how the motion can be transmitted by two toothed wheels, consider two plain circular wheels A and B mounted on shafts, having sufficient rough surfaces and pressing against each other

Let the wheel A be keyed to the rotating shaft and the wheel B to the shaft, to be rotated. A little consideration will show that when the wheel A is rotated by a rotating shaft, it will rotate the wheel B in the opposite direction. The wheel B will be rotated (by the wheel A) so long as the tangential force exerted by the wheel A does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (P) exceeds the *frictional resistance (F), slipping will take place between the two wheels. Thus the friction drive is not a positive drive.





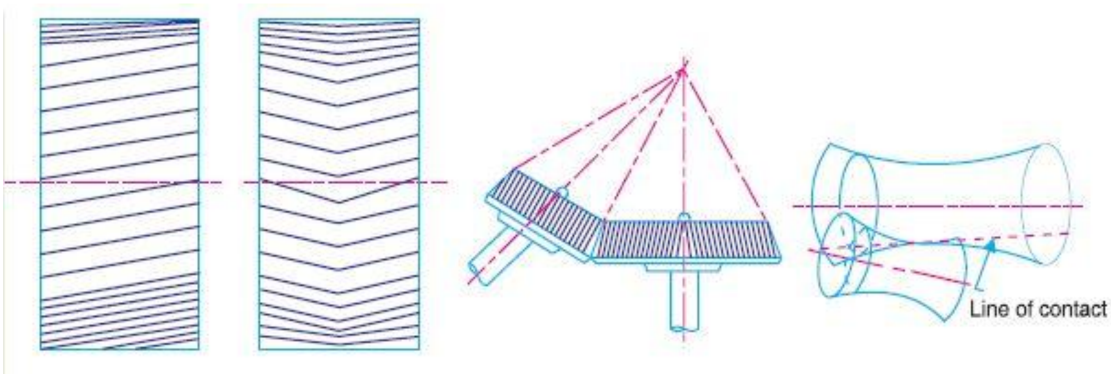
In order to avoid the slipping, a number of projections (called teeth) are provided on the periphery of the wheel A, which will fit into the corresponding recesses on the periphery of the wheel B. A friction wheel with the teeth cut on it is known as toothed wheel or gear. The usual connection to show the toothed wheels is by their pitch circles.

Classification of Toothed Wheels

1. According to the position of axes of the shafts. The axes of the two shafts between which the motion is to be transmitted, may be

(a) Parallel, (b) Intersecting, and (c) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears These gears are called spur gears and the arrangement is known as spur gearing. These gears have teeth parallel to the axis of the wheel as another name given to the spur gearing is helical gearing, in which the teeth are inclined to the axis. The single and double helical gears connecting parallel shafts are respectively. The double helical gears are known as herringbone gears. A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to parallel shafts and having a line contact. The two non-parallel or intersecting, but coplanar shafts connected by gears is shown in These gears are called bevel gears and the arrangement is known as bevel gearing. The bevel gears, like spur gears, may also have their teeth inclined to the face of the bevel, in which case they are known as helical bevel gears. The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears is shown in These gears are called skew bevel gears or spiral gears and the arrangement is known as skew bevel gearing or spiral gearing. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as hyperboloids.

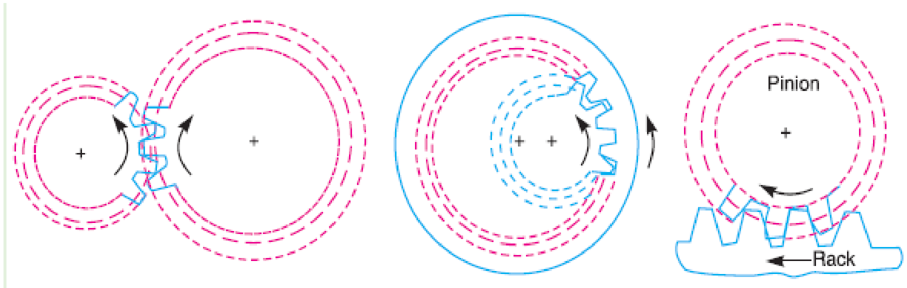


(*a*) Single helical gear. (*b*) Double helical gear. (*c*) Bevel gear. (*d*) Spiral gear.

2. According to the peripheral velocity of the gears. The gears, according to the peripheral velocity of the gears may be classified as :
(a) Low velocity, (b) Medium velocity, and (c) High velocity.
The gears having velocity less than 3 m/s are termed as low velocity gears and gears having velocity between 3 and 15 m/s are known as medium velocity gears. If the velocity of gears is more than 15 m/s, then these are called high speed gears.

3. According to the type of gearing. The gears, according to the type of gearing may be classified as :
(a) External gearing, (b) Internal gearing, and (c) Rack and pinion.
In external gearing, the gears of the two shafts mesh externally with each other as shown in Fig. 12.3 (a). The larger of these two wheels is called spur wheel and the smaller wheel is called pinion. In an external gearing, the motion of the two wheels is always unlike In internal gearing, the gears of the two shafts mesh internally with each other as shown in Fig. 12.3 (b). The larger of these two wheels is called annular wheel and the smaller wheel is called pinion. In an internal gearing, the motion of the two wheels is

always like Sometimes, the gear of a shaft meshes externally and internally with the gears in a straight line. Such type of gear is called rack and pinion. The straight line gear is called rack and the circular wheel is called pinion. A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and vice-versa



4. According to position of teeth on the gear surface. The teeth on the gear surface may be
(a) straight, (b) inclined, and (c) curved.
We have discussed earlier that the spur gears have straight teeth where as helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.

Condition for Constant Velocity Ratio of Toothed Wheels
Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 12.6. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure.

Let $T\ T$ be the common tangent and MN be the common normal to the curves at the point of contact Q. From the centres O_1 and O_2, draw O_1M and O_2N perpendicular to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.

Let v_1 and v_2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

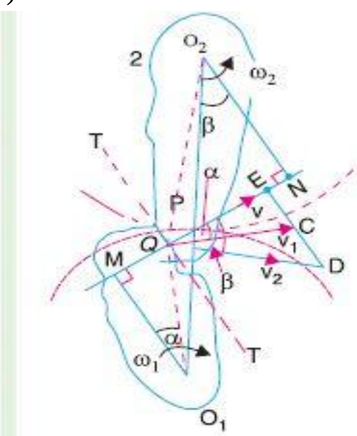


Fig. 12.6. Law of gearing.

$$\therefore \quad v_1 \cos\alpha = v_2 \cos\beta$$

or

$$(\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$$

$$(\omega_1 \times O_1Q)\frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q)\frac{O_2N}{O_2Q} \quad \text{or} \quad \omega_1 \times O_1M = \omega_2 \times O_2N$$

$$\therefore \quad \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \qquad \ldots(i)$$

Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad \ldots(ii)$$

Combining equations (*i*) and (*ii*), we have

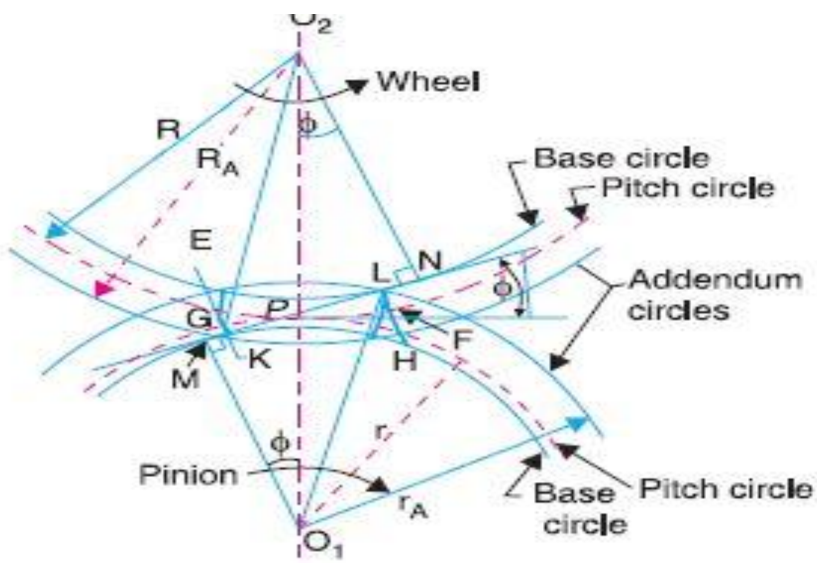
$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad \ldots(iii)$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres O1 and O2, or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities.

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels. In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point. This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of gearing.

Length of Path of Contact

Consider a pinion driving the wheel as shown in Fig. 12.11. When the pinion rotates in clockwise direction, the contact between a pair of in volute teeth begins at K (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and* ends at L (outer end of the tooth face on the pinion or on the flank near the base circle of wheel). MN is the common normal at the point of contacts and the common tangent to the base circles. The point K is the intersection of the addendum circle of wheel and the common tangent. The point L is the intersection of the addendum circle of pinion and Common tangent



We have discussed in Art. 12.4 that the length of path of contact is the length of common normal cutoff by the addendum circles of the wheel and the pinion. Thus the length of path of contact is KL which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as path of approach and the part of the path of contact PL is known as path of recess.

Let $r_A = O_1L =$ Radius of addendum circle of pinion,

$R_A = O_2K =$ Radius of addendum circle of wheel,

$r = O_1P =$ Radius of pitch circle of pinion, and

$R = O_2P =$ Radius of pitch circle of wheel.

From Fig. 12.11, we find that radius of the base circle of pinion,

$$O_1M = O_1P \cos \phi = r \cos \phi$$

and radius of the base circle of wheel,

$$O_2N = O_2P \cos \phi = R \cos \phi$$

Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2 \cos^2 \phi}$$

and $$PN = O_2P \sin \phi = R \sin \phi$$

$\therefore$ Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

Similarly from right angled triangle O_1ML,

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2 \cos^2 \phi}$$

and $$MP = O_1P \sin \phi = r \sin \phi$$

$\therefore$ Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$\therefore$ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} + \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi$$

Length of Arc of Contact

We have already defined that the arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Fig. 12.11, the arc of contact is EPF or GPH. Considering the arc of contact GPH, it is divided into two parts i.e. arc GP and arc PH. The arc GP is known as arc of approach and the arc PH is called arc of recess. The angles subtended by these arcs at O1 are called angle of approach and angle of recess respectively.

We know that the length of the arc of approach (arc GP)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

and the length of the arc of recess (arc PH)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

Since the length of the arc of contact GPH is equal to the sum of the length of arc of approach and arc of recess, therefore,

Length of the arc of contact

$$= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch. Mathematically,

Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of the arc of contact}}{p_c}$$

where p_c = Circular pitch = πm, and

m = Module.

Gear Trains

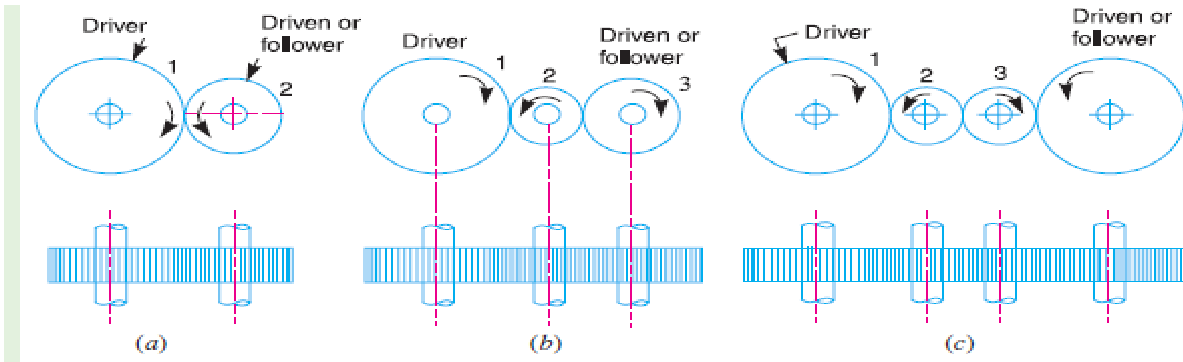
Introduction:

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels. The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains:

1. Simple gear train, 2. Compound gear train, 3. Reverted gear train, and 4. Epicyclic gear train.

Simple Gear Train: When there is only one gear on each shaft, it is known as simple gear train. The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. 13.1 (a). Since the gear 1 drives the gear 2, therefore gear 1 is called the driver and the gear 2 is called the driven or follower. It may be noted that the motion of the driven gear is opposite to the motion of driving gear.



Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

From above, we see that the train value is the reciprocal of speed ratio.Sometimes, the distance between the two gears is large. The motion from one gear to another,in such a case, may be transmitted by either of the following two methods :

1. By providing the large sized gear, or

2. By providing one or more intermediate gears.

A little consideration will show that the former method (i.e. providing large sized gears) is very inconvenient and uneconomical method ; whereas the latter method (i.e. providing one or more intermediate gear) is very convenient and economical. It may be noted that when the number of intermediate gears are odd, the motion of both the gears (i.e. driver and driven or follower) is like as But if the number of intermediate gears are even, the motion of the driven or follower will be in the opposite direction of the driver   Now consider a simple train of gears with one intermediate gear

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{...}(i)$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \quad \text{...}(ii)$$

The speed ratio of the gear train as shown in Fig. 13.1 (*b*) is obtained by multiplying the equations (*i*) and (*ii*).

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \quad \text{or} \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

i.e.

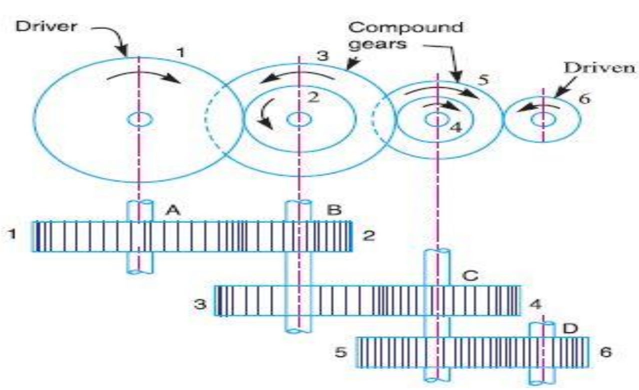
$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

and

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

1. Compound Gear Train

When there are more than one gear on a shaft it is called a compound train of gear. We have seen that the idle gears, in a simple train of gears do not effect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven. But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft



In a compound train of gears, as shown in Fig. 13.2, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \ldots(i)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \qquad \ldots(ii)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \qquad \ldots(iii)$$

The speed ratio of compound gear train is obtained by multiplying the equations (*i*), (*ii*) and (*iii*),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^*\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}} = \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}} = \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

Epicyclic Gear Train

We have already discussed that in an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. 13.6, where a gear A and the arm C have a common axis at O 1 about which they can rotate.

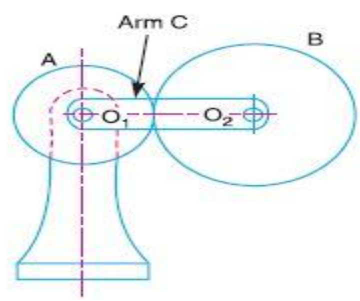
We know that speed ratio $= \frac{\text{Speed of first driver}}{\text{Speed of last driven}} = \frac{N_A}{N_D} = 12$

Also $\frac{N_A}{N_D} = \frac{N_A}{N_B} \times \frac{N_C}{N_D}$ ...($N_B = N_C$, being on the same shaft)

For $\frac{N_A}{N_B}$ and $\frac{N_C}{N_D}$ to be same, each speed ratio should be $\sqrt{12}$ so that

$$\frac{N_A}{N_D} = \frac{N_A}{N_B} \times \frac{N_C}{N_D} = \sqrt{12} \times \sqrt{12} = 12$$

The gear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear Bor vice- versa, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate upon and around gear A. Such a motion is called epicyclic and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as epicyclic gear trains (epi. means upon and cyclic means around). The epicyclic gear trains may be simple or compound. The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.



Velocity Ratios of Epicyclic Gear Train

1. Tabular method. Consider an epicyclic gear train as shown in Fig. 13.6.

Let T_A = Number of teeth on gear A, and

T_B = Number of teeth on gear B.

First of all, let us suppose that the arm is fixed. Therefore the axes of both the gears are also fixed relative to each other. When the gear A makes one revolution anticlockwise, the gear B will make T_A / T_B revolutions, clockwise Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear A makes + 1 revolution, then the gear B will make($- T_A / T_B$) revolutions. This statement of relative motion is entered in the first row of the table (see Table 13.1). Secondly, if the gear

A makes + x revolutions, then the gear B will make – x × T A / T B revolutions. This statement is entered in the second row of the table. In other words, multiply the each motion (entered in the first row) by x.

Thirdly, each element of an epicyclic train is given + y revolutions and entered in the third row. Finally, the motion of each element of the gear train is added up and entered in the fourth row.

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through $+ x$ revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add $+ y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

A little consideration will show that when two conditions about the motion of rotation of any two elements are known, then the unknown speed of the third element may be obtained by substituting the given data in the third column of the fourth row